

EE 330

Lecture 15

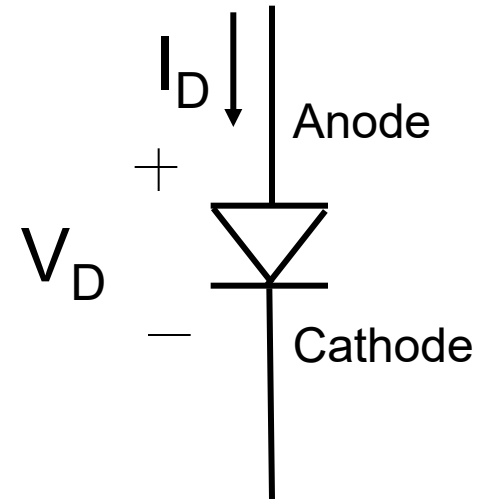
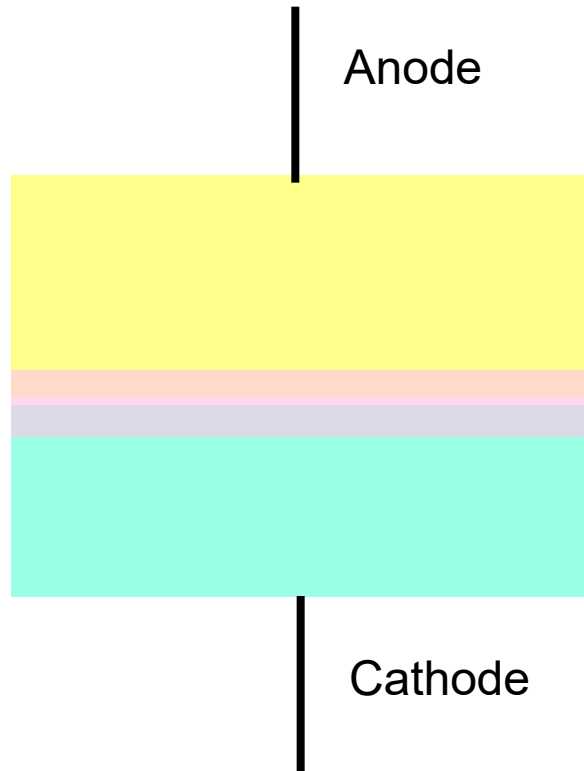
Devices in Semiconductor Processes

- Diodes
- Analysis of Nonlinear Circuits
- Capacitor Models

Fall 2025 Exam Schedule

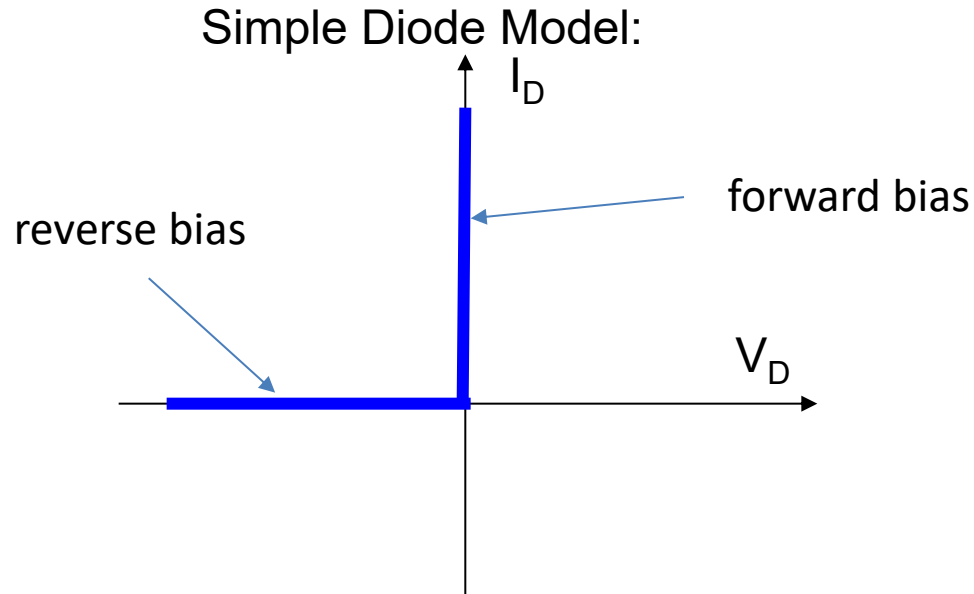
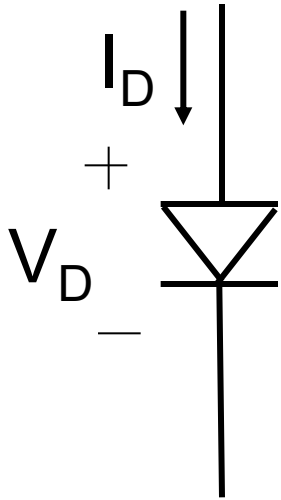
Exam 1	Friday	Sept 26
Exam 2	Friday	October 24
Exam 3	Friday	Nov 21
Final Exam	Monday	Dec 15 12:00 - 2:00 PM

pn Junctions



Circuit Symbol

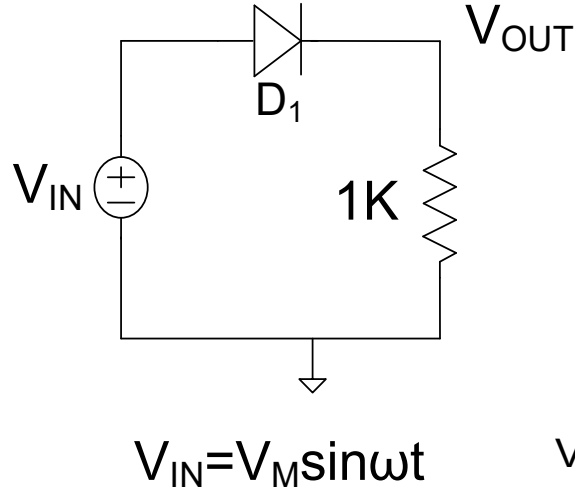
pn Junctions



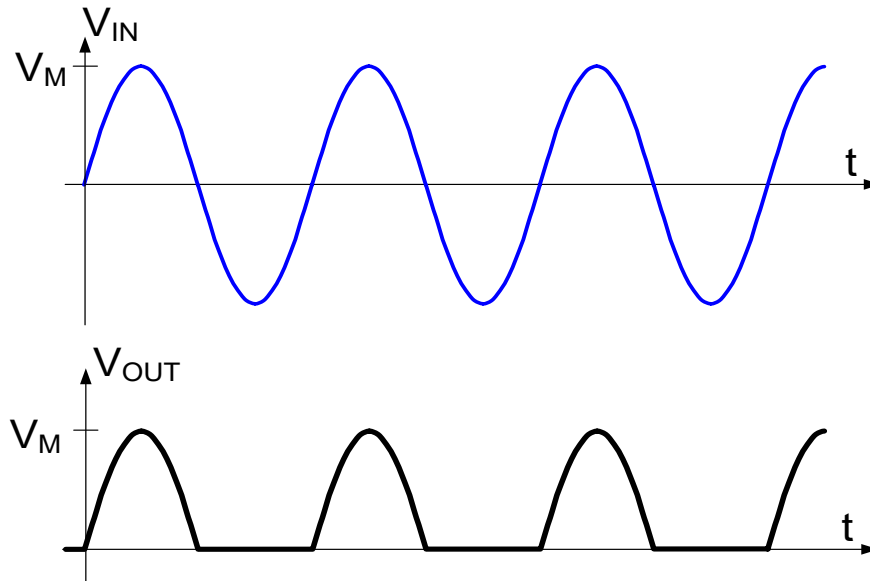
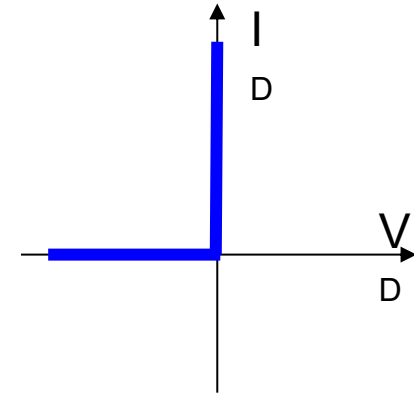
- This is a piecewise model
- pn junction serves as a “rectifier” passing current in one direction and blocking it in the other direction

Review from last lecture

Rectifier Application:



Simple Diode Model:

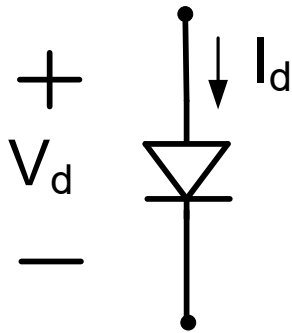


Analysis based upon “passing current” in one direction and “blocking current” in the other direction

I-V characteristics of pn junction

(signal or rectifier diode)

Improved Diode Model:



I_S in the 10fA to 100fA range

I_S proportional to junction area

$$V_t = \frac{kT}{q}$$

$$k = 1.380\,64852 \times 10^{-23} \text{ JK}^{-1}$$

$$q = -1.60217662 \times 10^{-19} \text{ C}$$

$$k/q = 8.62 \times 10^{-5} \text{ VK}^{-1}$$

n typically about 1

Diode equation due to William Shockley, inventor of BJT

In 1919, [William Henry Eccles](#) coined the term **diode**

In 1940, Russell Ohl “stumbled upon” the p-n junction diode

Diode Equation

$$I_D = I_S \left(e^{\frac{V_d}{nV_t}} - 1 \right)$$

I_S and n are model parameters

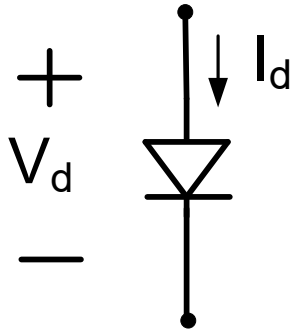
What is V_t at room temp?

V_t is about 26mV at room temp

I-V characteristics of pn junction

(signal or rectifier diode)

Improved Diode Model:

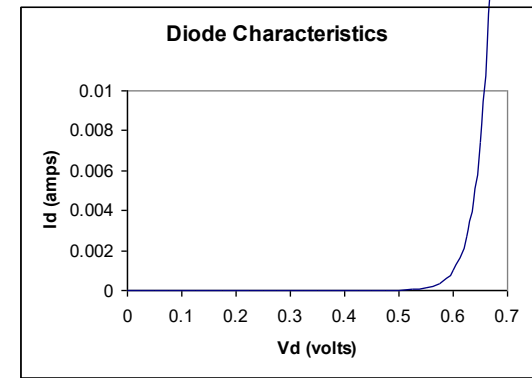


Diode Equation $I_D = I_S \left(e^{\frac{V_d}{nV_t}} - 1 \right)$
 (not a piecewise model !)

Simplification of Diode Equation:

Under reverse bias ($V_d < 0$), $I_D \cong -I_S$

Under forward bias ($V_d > 0$), $I_D = I_S e^{\frac{V_d}{nV_t}}$



I_S in 10fA - 100fA range (for signal diodes)

n typically about 1

$$V_t = \frac{kT}{q}$$

$$k/q = 8.62 \times 10^{-5} \text{ VK}^{-1}$$

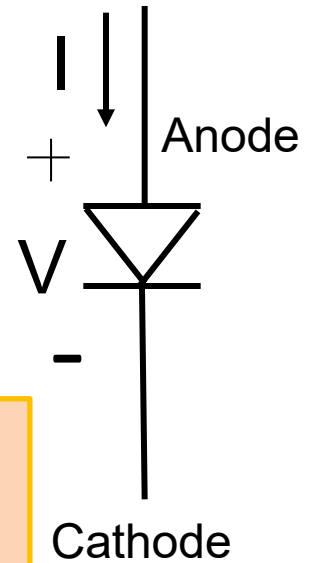
V_t is about 26mV at room temp

Simplification essentially identical model except for V_d very close to 0

Diode Equation or forward bias simplification are unwieldy to work with analytically

Diode Model Summary

Ideal Diode Model	$V_D = 0$	$I_D > 0$	forward bias
	$I_D = 0$	$V_D < 0$	reverse bias



$$I_S = J_S A$$

Diode Equation

$$I_D = I_S \left(e^{\frac{V_d}{nV_t}} - 1 \right)$$

Diode Equation: (simplification)

$$I = \begin{cases} I_S e^{\frac{V}{nV_T}} & V > 0 \\ -I_S & V < 0 \end{cases}$$

forward bias
reverse bias

Diode Equation: (further simplification)

$$I = \begin{cases} I_S e^{\frac{V}{nV_T}} & V > 0 \\ 0 & V < 0 \end{cases}$$

forward bias
reverse bias

Little difference in these models, if any, in most applications. Typically, any referred to as the Diode Equation

pn Junctions

Diode Equation: (further simplification)

$$I = \begin{cases} J_s A e^{\frac{V}{nV_T}} & V > 0 \text{ forward bias} \\ 0 & V < 0 \text{ reverse bias} \end{cases}$$

$$I_s = J_s A$$

J_s (or I_s) is strongly temperature dependent

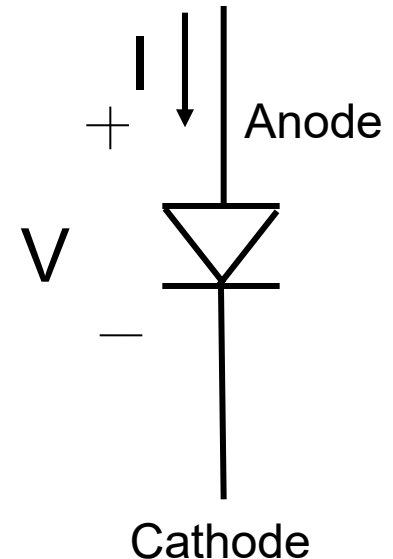
With $n=1$, for $V>0$,

$$J_s = J_{sx} T^m e^{\frac{-V_{G0}}{V_t}}$$

$\{J_{sx}, m, n\}$ are model parameters

$\{A\}$ is a design parameter

$\{T, V_{G0}, k/q\}$ are environmental parameters and physical constants



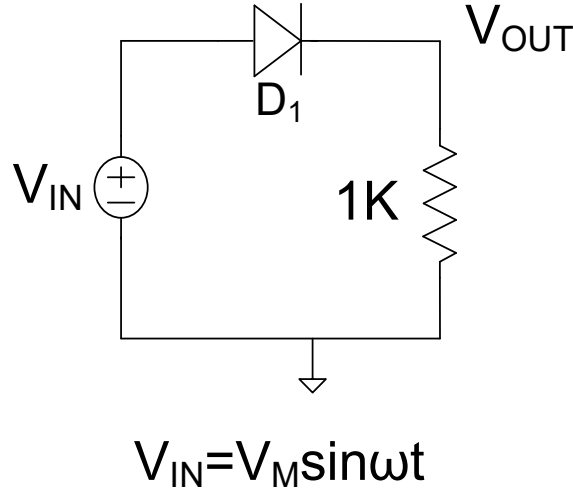
Diode Equation: (further simplification showing more detail)

$$I(T) = \begin{cases} \left(J_{sx} \left[T^m e^{\frac{-V_{G0}}{V_t}} \right] \right) A e^{\frac{V}{V_t}} & V > 0 \\ 0 & V < 0 \end{cases}$$

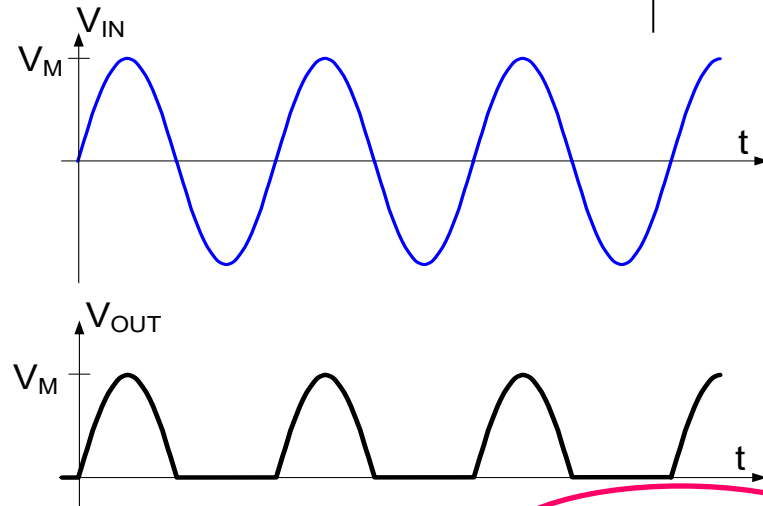
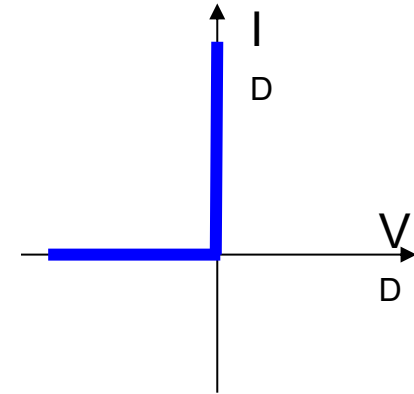
Typical values for key parameters: $J_{sx}=0.5\text{A}/\mu^2$, $V_{G0}=1.17\text{V}$, $m=2.3$

Observe this simplification is a piecewise model !

Rectifier Application:



Simple Diode Model:



Analysis based upon “passing current” in one direction and “blocking current” in the other direction

What principle was used in this analysis?

Was this analysis rigorous ?

Diode Equation (even simplification) unwieldy to work with analytically. **Why?**

World's simplest diode circuit

Determine V_{OUT}

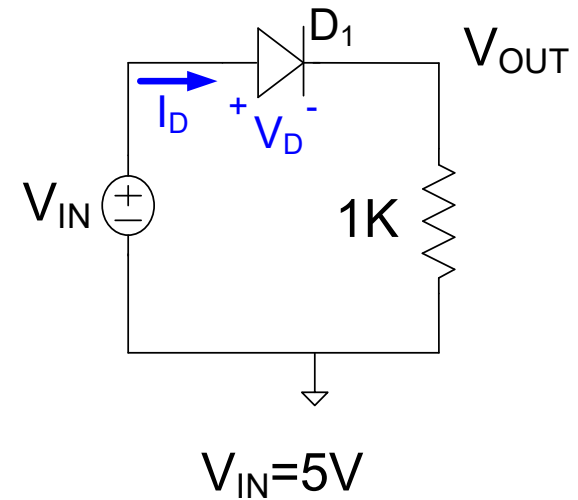
Assume forward bias, simplified diode equation model

$$\left. \begin{aligned} 5 &= V_D + V_{OUT} \\ V_{OUT} &= I_D \cdot 1K \\ I_D &= I_S e^{\frac{V_D}{nV_t}} \end{aligned} \right\}$$

3 independent equations and 3 unknowns



$$V_{OUT} = I_S e^{\frac{5-V_{OUT}}{nV_t}} \cdot 1K$$
$$V_{OUT}=?$$

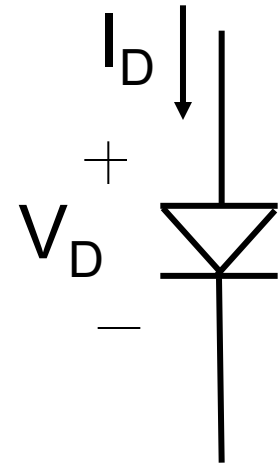


- Can obtain V_{OUT} from this equation but explicit expression does not exist for V_{OUT} !
- Previous analysis based upon “passing” and “blocking” currents was not rigorous !!

I_S highly temperature dependent

Example: Consider diode operating under forward bias

$$I_D(T) = \left(J_{SX} \left[T^m e^{\frac{-V_{G0}}{V_t}} \right] \right) A e^{\frac{V_D}{V_t}}$$



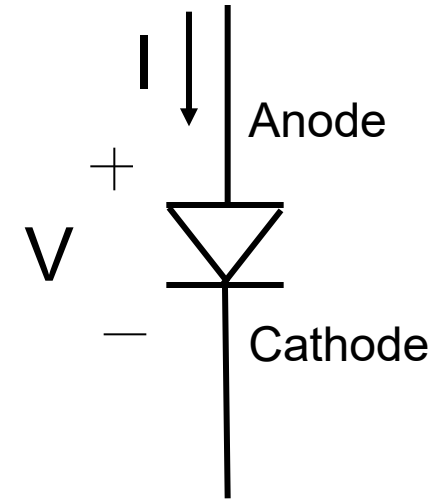
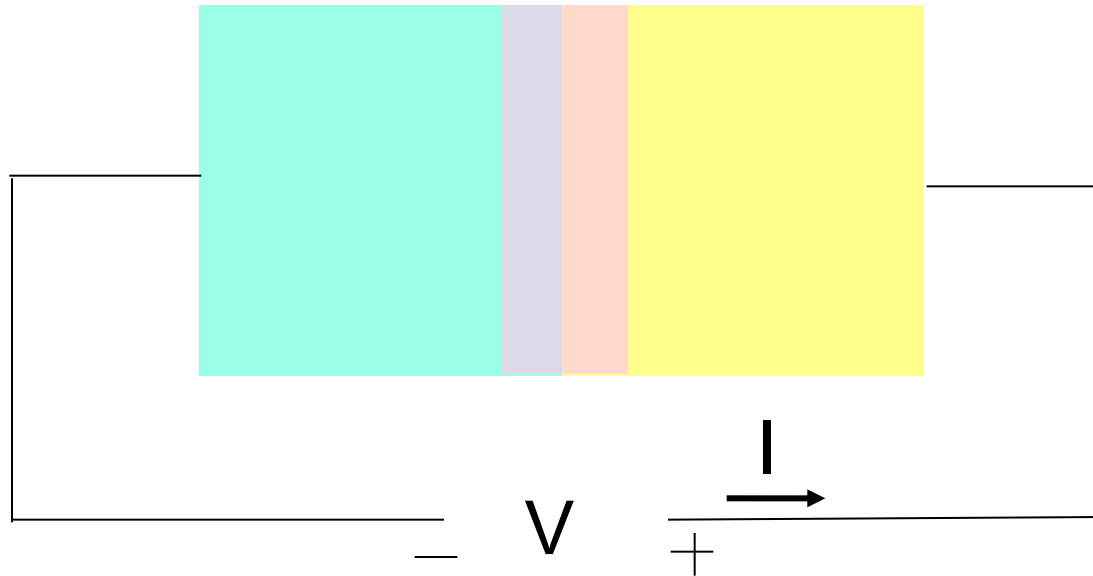
What percent change in I_S will occur for a 1°C change in temperature at room temperature?

$$\frac{\Delta I_S}{I_S} = \frac{\left(J_{SX} \left[T_{T_2}^m e^{\frac{-V_{G0}}{V_t(T_2)}} \right] \right) A - \left(J_{SX} \left[T_{T_1}^m e^{\frac{-V_{G0}}{V_t(T_1)}} \right] \right) A}{\left(J_{SX} \left[T_{T_1}^m e^{\frac{-V_{G0}}{V_t(T_1)}} \right] \right) A} = \frac{\left(\left[T_{T_2}^m e^{\frac{-V_{G0}}{V_t(T_2)}} \right] \right) - \left(\left[T_{T_1}^m e^{\frac{-V_{G0}}{V_t(T_1)}} \right] \right)}{\left(\left[T_{T_1}^m e^{\frac{-V_{G0}}{V_t(T_1)}} \right] \right)}$$

$$\frac{\Delta I_S}{I_S} = \frac{(1.240 \times 10^{-15}) - (1.025 \times 10^{-15})}{(1.025 \times 10^{-15})} 100\% = 21\%$$

- Attempts to measure I_S in our laboratories can result in large errors !
- Most circuits whose performance depends upon precise value for I_S are not practical

pn Junctions

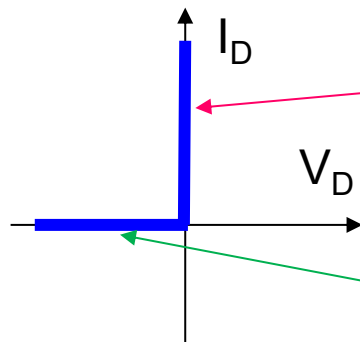


Diode Equation:
(good enough for most applications)

$$I = \begin{cases} J_s A e^{\frac{V}{nV_T}} & V > 0 \\ 0 & V < 0 \end{cases}$$

$$I_s = J_s A$$

Simple Diode Model:

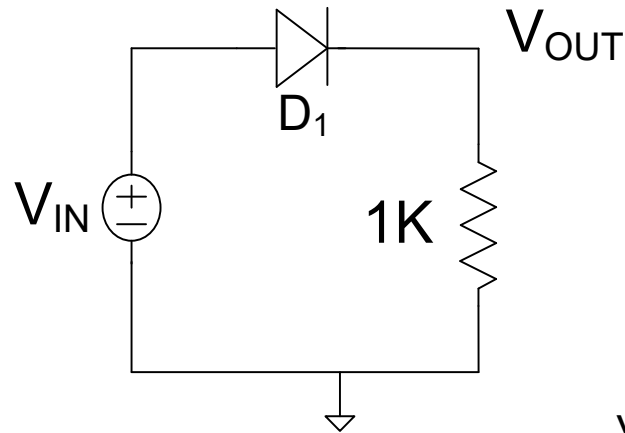


Often termed the “conducting” or “ON” state

Often termed the “nonconducting” or “OFF” state

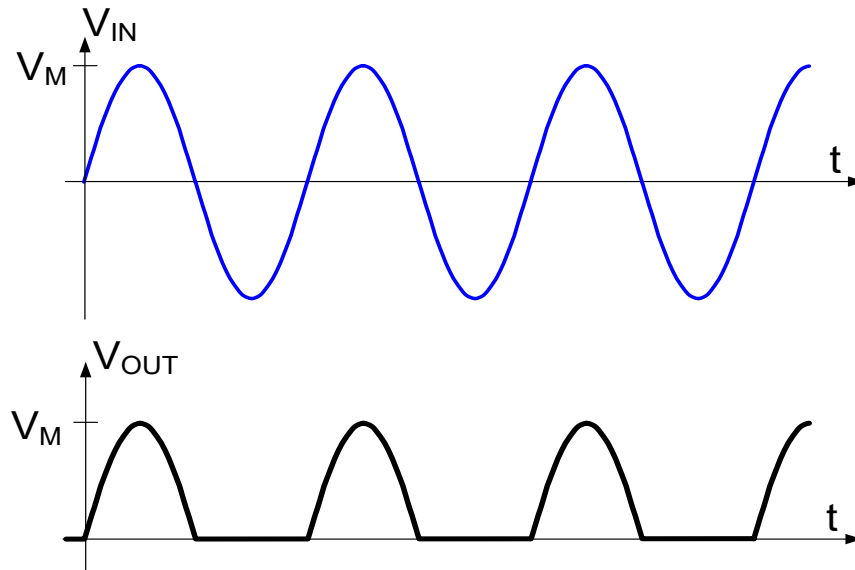
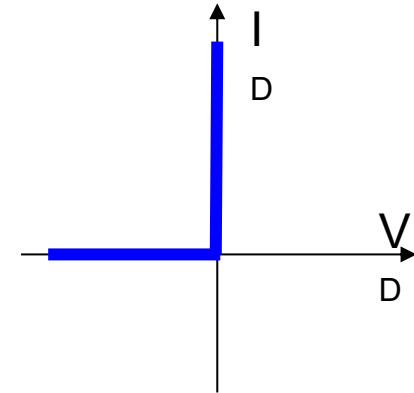
What basic circuit analysis principles were used to analyze this circuit?

Rectifier Application:



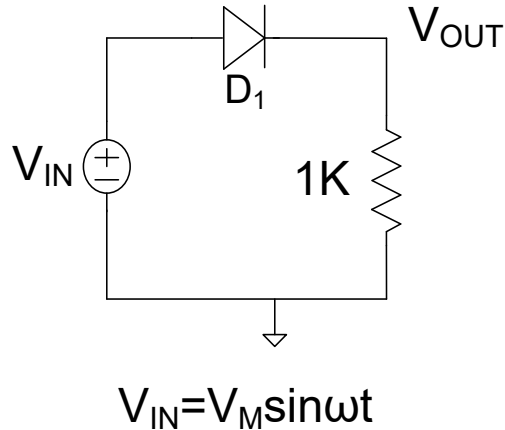
$$V_{IN} = V_M \sin \omega t$$

Simple Diode Model:

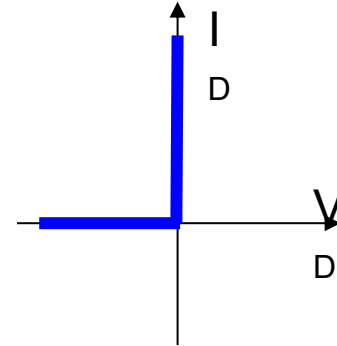


Analysis based upon “passing current” in one direction and “blocking current” in the other direction

Rectifier Application:



Simple Diode Model:



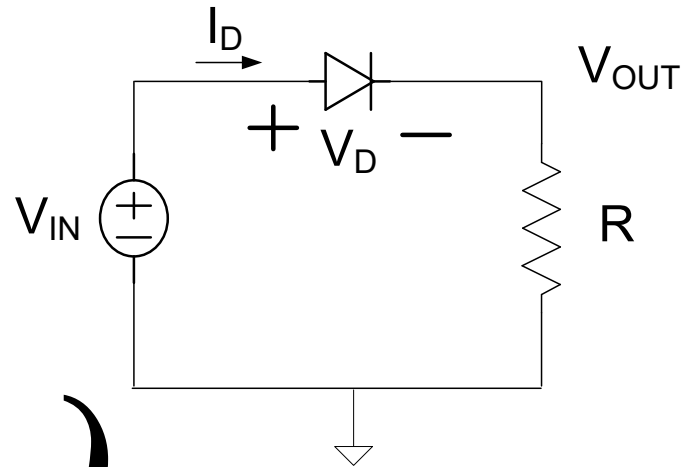
Analysis based upon “passing current” in one direction and “blocking current” in the other direction

Was the previous analysis rigorous?

Is use of simple diode model justifiable?

Review from last lecture

Consider again the basic rectifier circuit



$$V_{IN} = V_D + I_D R$$

$$V_{OUT} = I_D R$$

$$I_D = I_S \left(e^{\frac{V_D}{V_t}} - 1 \right)$$

$$V_{OUT} = I_S R \left(e^{\frac{V_{IN} - V_{OUT}}{V_t}} - 1 \right)$$

This analysis is rigorous (using only KVL and device models)

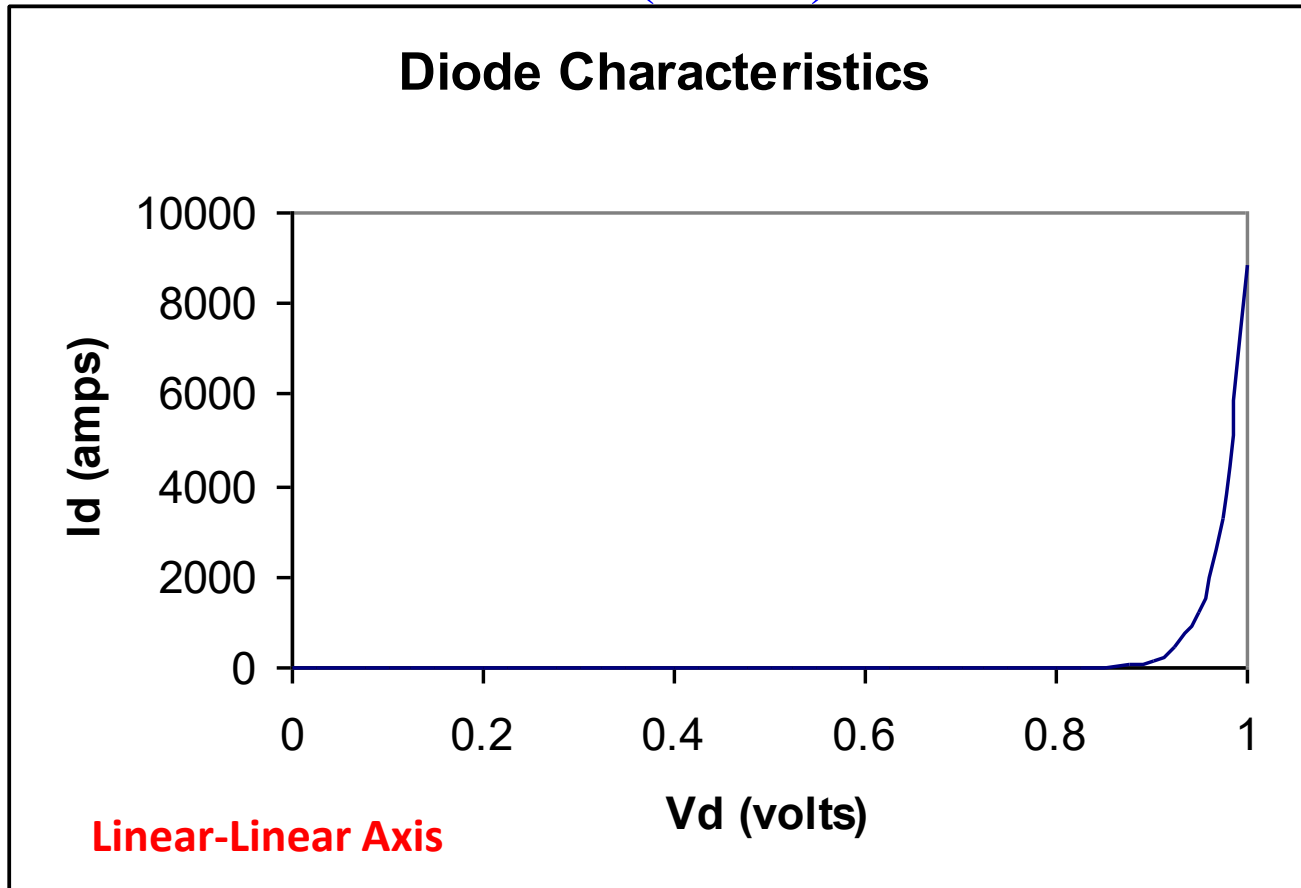
Even the simplest diode circuit does not have a closed-form explicit solution when diode equation is used to model the diode !!

Due to the nonlinear nature of the diode equation

Simplifications of diode model are essential if analytical results are to be obtained !

Lets study the diode equation a little further

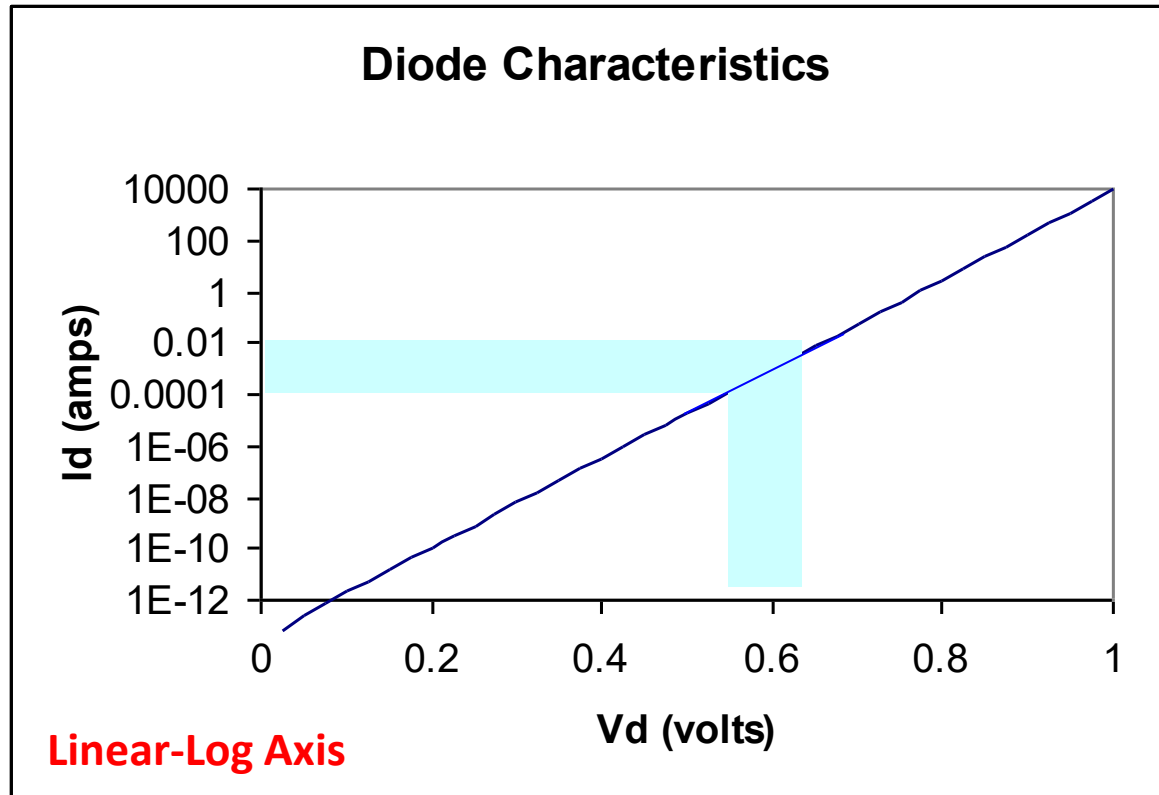
$$I_d = I_s \left(e^{\frac{V_d}{V_t}} - 1 \right)$$



Power Dissipation Becomes Destructive if $V_d > 0.85V$ (actually less)

Lets study the diode equation a little further

$$I_d = I_s \left(e^{\frac{V_d}{V_t}} - 1 \right)$$

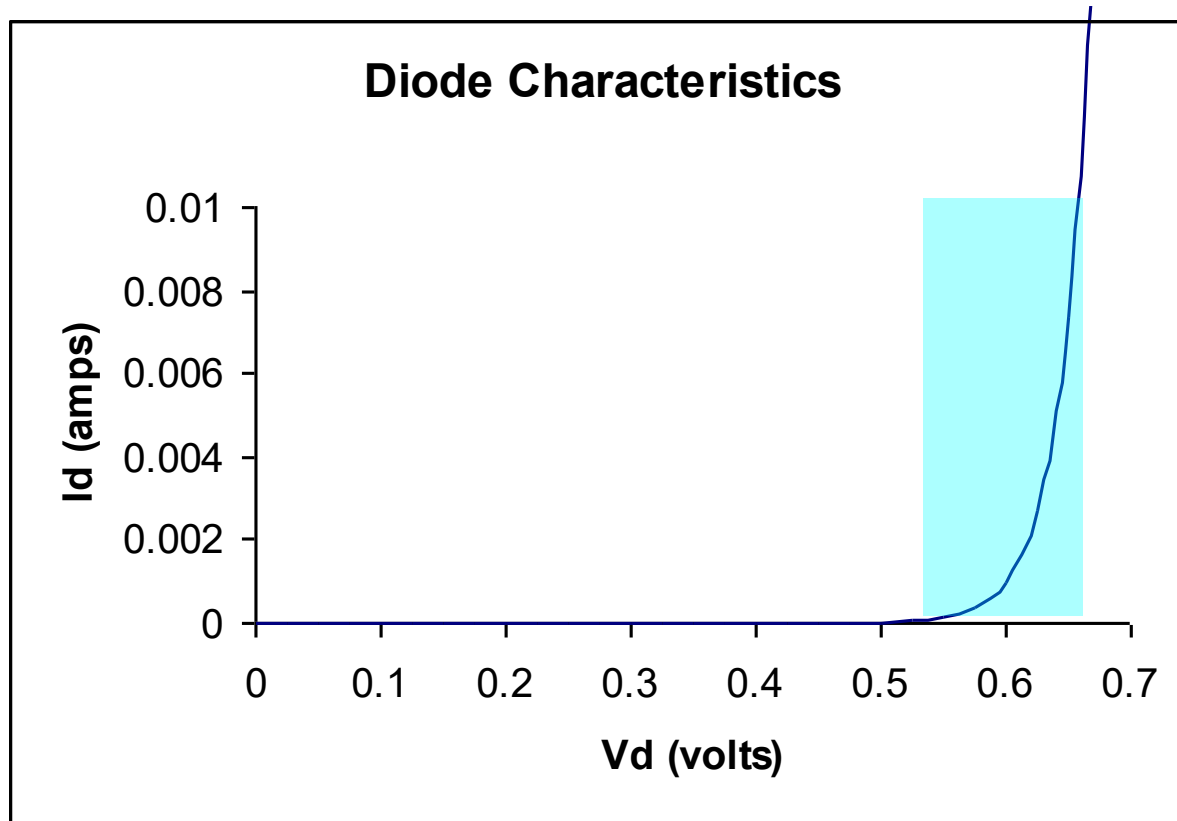


For two decades of current change, V_d is close to 0.6V

This is the most useful conducting current range for many applications

Lets study the diode equation a little further

$$I_d = I_s \left(e^{\frac{V_d}{V_t}} - 1 \right)$$

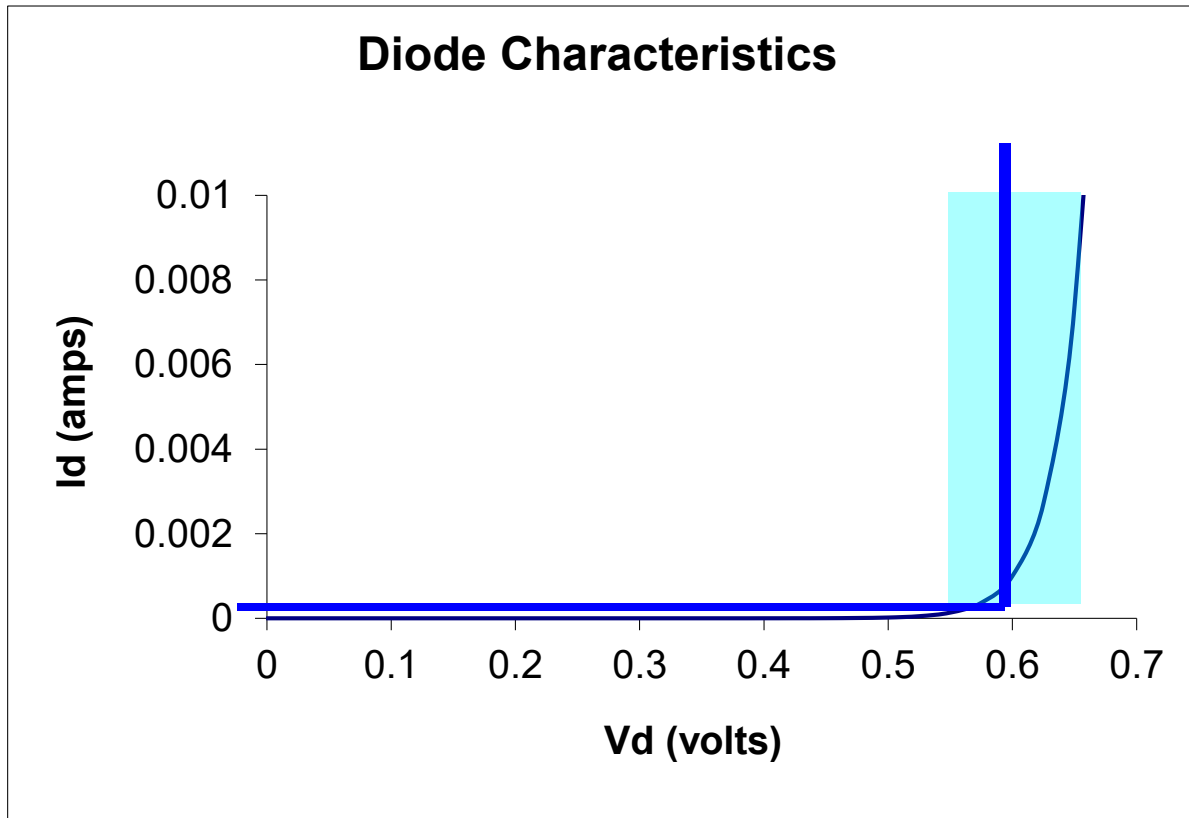


For two decades of current change, V_d is close to 0.6V

This is the most useful current range when conducting for many applications

Lets study the diode equation a little further

$$I_d = I_s \left(e^{\frac{V_d}{V_t}} - 1 \right) \quad \longrightarrow \quad \begin{array}{ll} I_d = 0 & \text{if } V_d < 0.6 \text{ V} \\ V_d = 0.6 \text{ V} & \text{if } I_d > 0 \end{array}$$



Widely Used Piecewise Linear Model

Note the “if” conditions in this model !

Which simplified model is better?

Both models were developed w/o specific application and are about the same !

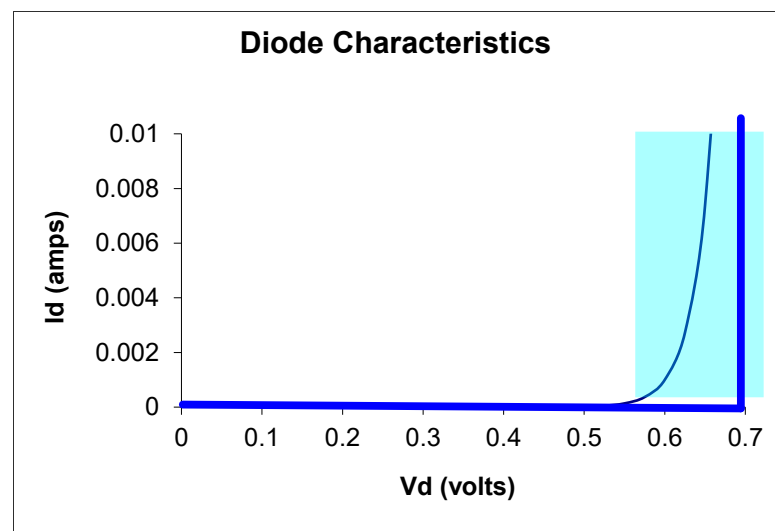
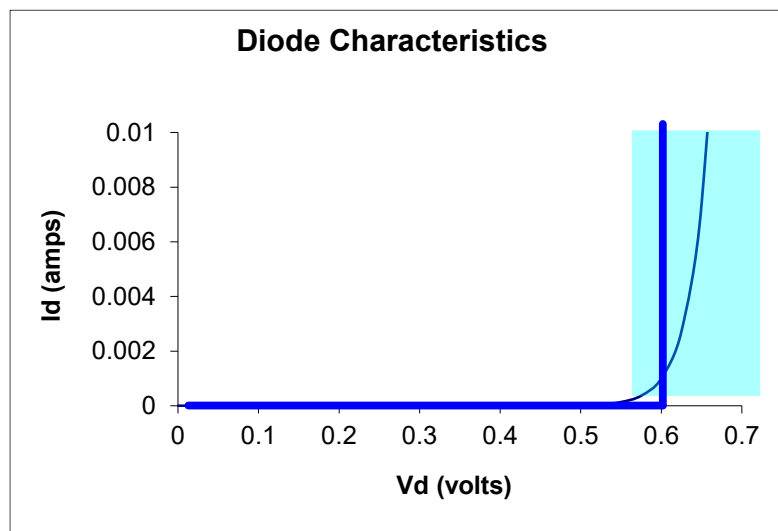
$$I_d = I_s \left(e^{\frac{V_d}{V_t}} - 1 \right)$$

$$I_d = 0 \quad \text{if} \quad V_d < 0.6 \text{ V}$$

$$V_d = 0.6 \text{ V} \quad \text{if} \quad I_d > 0$$

$$I_d = 0 \quad \text{if} \quad V_d < 0.7 \text{ V}$$

$$V_d = 0.7 \text{ V} \quad \text{if} \quad I_d > 0$$



Widely Used Piecewise Linear Model

Note the “if” conditions in this model !

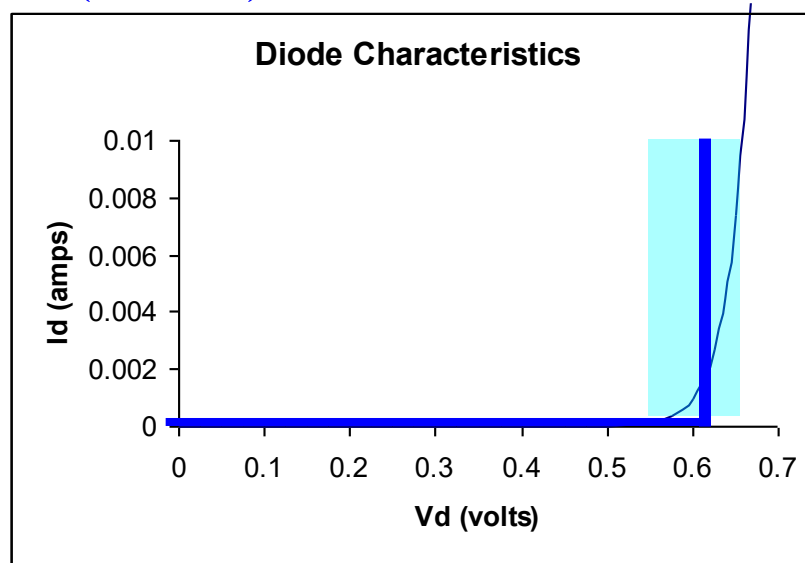
Lets study the diode equation a little further

$$I_d = I_s \left(e^{\frac{V_d}{V_t}} - 1 \right)$$

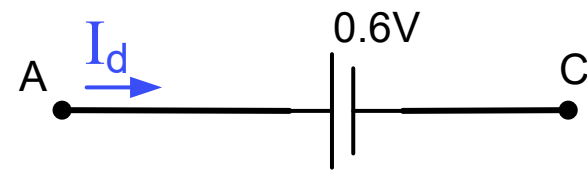
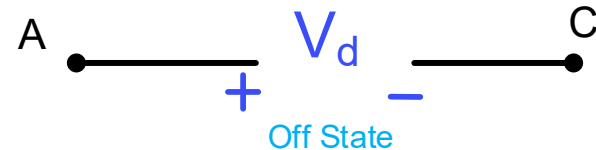
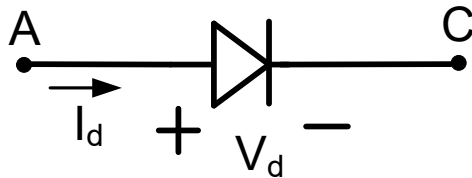


$$I_d = 0 \quad \text{if} \quad V_d < 0.6 \text{ V}$$

$$V_d = 0.6 \text{ V} \quad \text{if} \quad I_d > 0$$

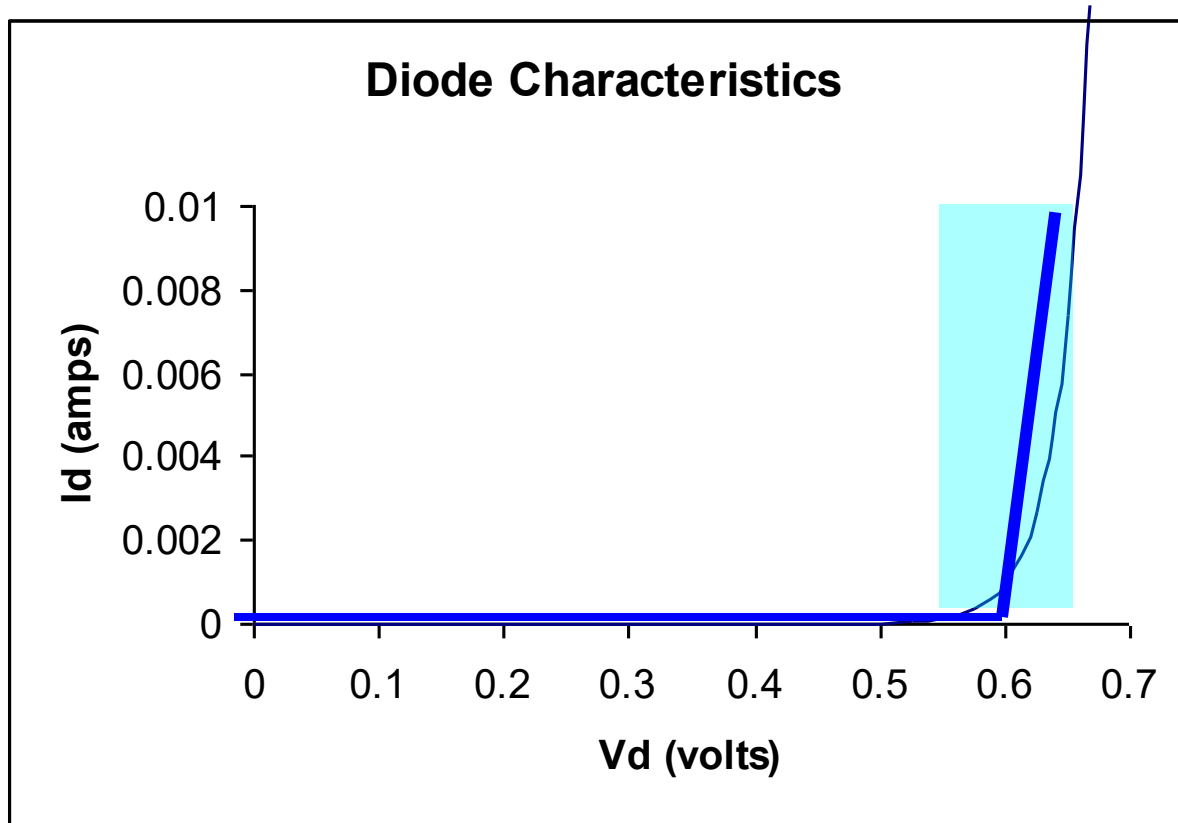


Equivalent Circuit



Lets study the diode equation a little further

$$I_d = I_s \left(e^{\frac{V_d}{V_t}} - 1 \right)$$



Better model in “ON” state though often not needed

Includes Diode “ON” resistance

Lets study the diode equation a little further

$$I_d = I_S \left(e^{\frac{V_d}{V_t}} - 1 \right)$$

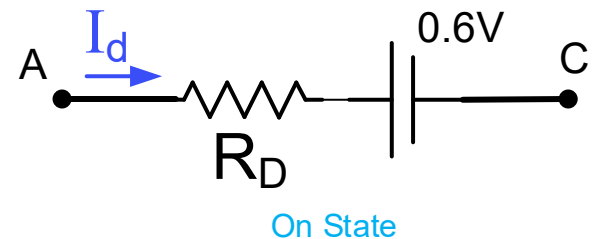
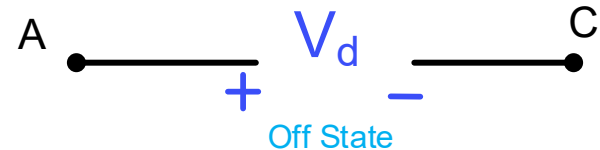
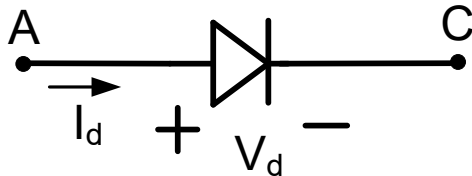
Piecewise Linear Model with Diode Resistance

$$I_d = 0 \quad \text{if } V_d < 0.6V$$

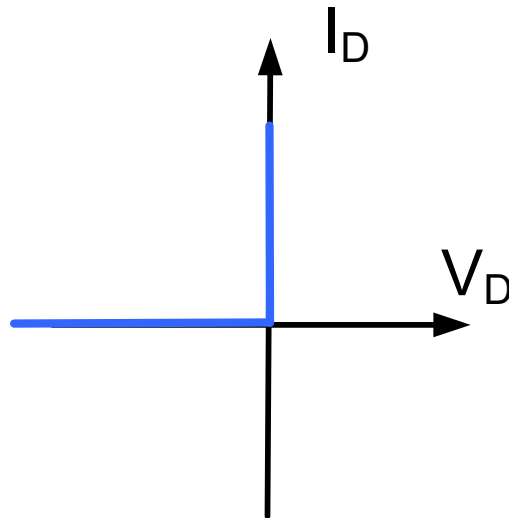
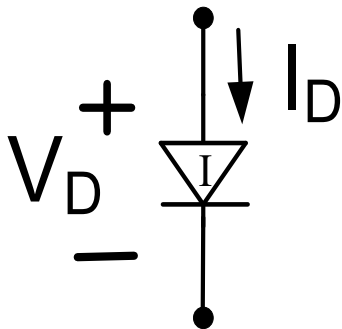
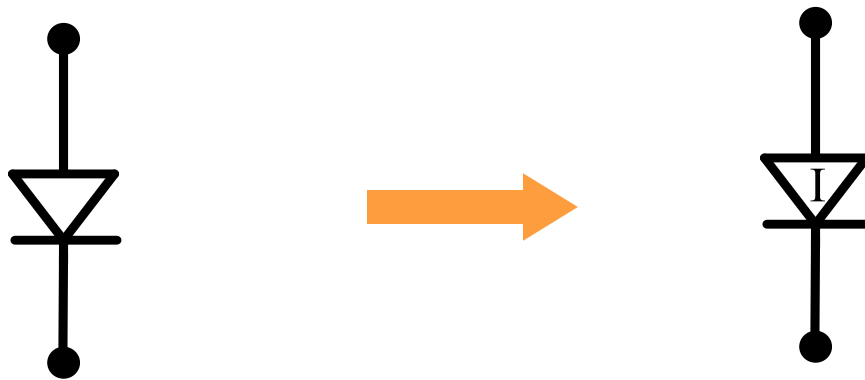
$$V_d = 0.6V + I_d R_D \quad \text{if } I_d > 0$$

(R_D is rather small: often in the 20Ω to 100Ω range):

Equivalent Circuit

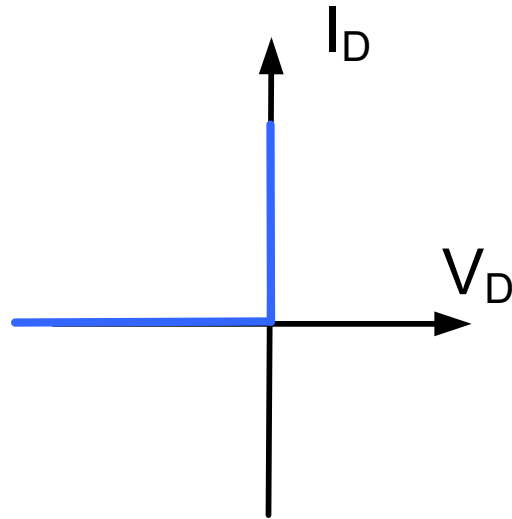


The Ideal Diode



$$\begin{aligned} I_D &= 0 & \text{if } V_D &\leq 0 \\ V_D &= 0 & \text{if } I_D &> 0 \end{aligned}$$

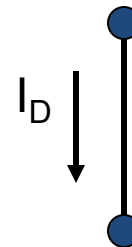
The Ideal Diode



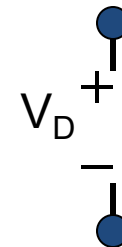
$$\begin{aligned} I_D &= 0 & \text{if } V_D &\leq 0 & \text{"OFF"} \\ V_D &= 0 & \text{if } I_D &> 0 & \text{"ON"} \end{aligned}$$



"ON"



"OFF"



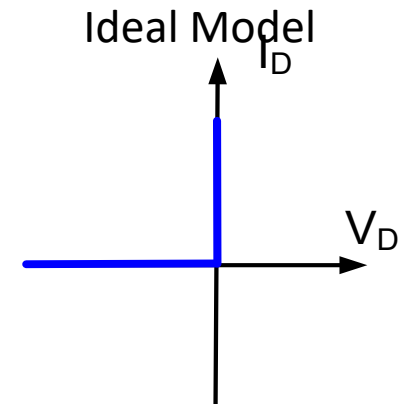
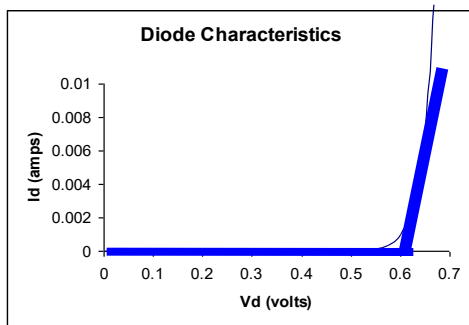
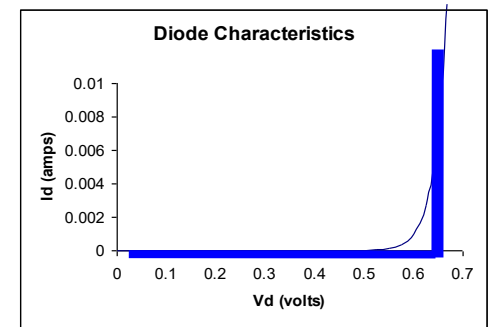
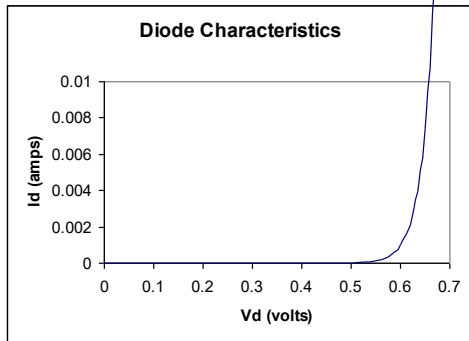
Valid for

$$I_D > 0$$

$$V_D \leq 0$$

Diode Models

Diode Equation (4 variants)



Which model should be used?

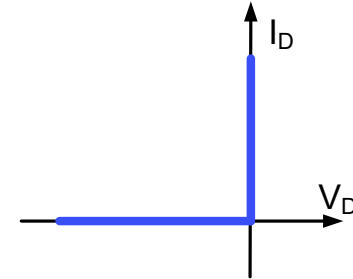
The simplest model that will give acceptable results in the analysis of a circuit

Diode Model Summary

Piecewise Linear Models

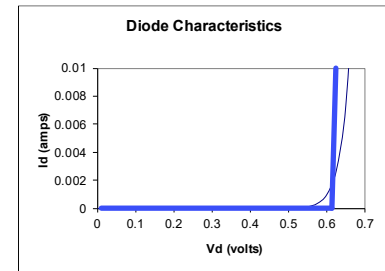
$$I_d = 0 \quad \text{if } V_d < 0$$

$$V_d = 0 \quad \text{if } I_d > 0$$



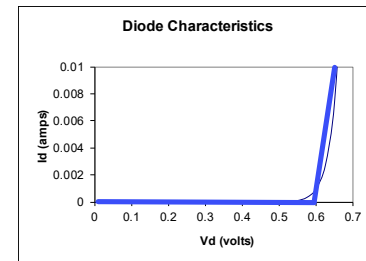
$$I_d = 0 \quad \text{if } V_d < 0.6V$$

$$V_d = 0.6V \quad \text{if } I_d > 0$$



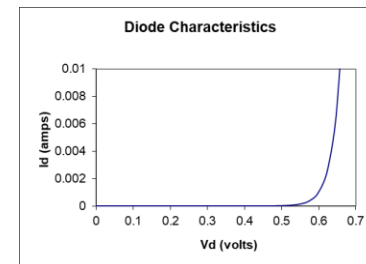
$$I_d = 0 \quad \text{if } V_d < 0.6$$

$$V_d = 0.6 + I_d R_d \quad \text{if } I_d > 0$$



Diode Equation (or variants discussed)

$$I_d = I_s \left(e^{\frac{V_d}{V_t}} - 1 \right)$$



Diode Model Summary

Piecewise Linear Models

$$I_d = 0 \quad \text{if } V_d < 0$$

$$V_d = 0 \quad \text{if } I_d > 0$$

$$I_d = 0 \quad \text{if } V_d < 0.6V$$

$$V_d = 0.6V \quad \text{if } I_d > 0$$

$$I_d = 0 \quad \text{if } V_d < 0.6$$

$$V_d = 0.6 + I_d R_d \quad \text{if } I_d > 0$$

Diode Equation (or variants discussed)

$$I_d = I_s \left(e^{\frac{V_d}{V_t}} - 1 \right)$$

When is the ideal model adequate?

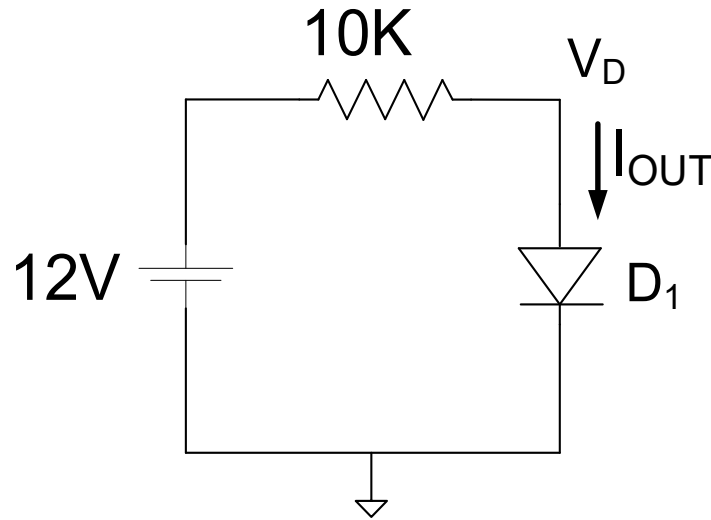
When it doesn't make much difference whether $V_d = 0V$ or $V_d = 0.6V$

When is the second piecewise-linear model adequate?

When it doesn't make much difference whether $V_d = 0.6V$ or $V_d = 0.7V$

Example:

Determine I_{OUT} for the following circuit



Solution:

If the diode equation model is used will obtain:

$$\left. \begin{aligned} 12 &= I_{OUT} \cdot 10K + V_D \\ I_{OUT} &= I_S \left(e^{\frac{V_D}{V_t}} - 1 \right) \end{aligned} \right\} \Rightarrow I_{OUT} = I_S \left(e^{\frac{-I_{OUT} \cdot 10K}{V_t}} e^{\frac{12}{V_t}} - 1 \right)$$

As in previous example, a closed-form explicit expression for I_{OUT} does not exist

Will now establish rigorous approach for solving this (and other) nonlinear circuit (with model uncertainty and piecewise models) with piecewise models and obtaining a practical solution !

Devices in Semiconductor Processes

- Resistors
- Diodes
- Capacitors
- MOSFETs



Side Track!
Analysis of Nonlinear Circuits

Analysis of Nonlinear Circuits

(Circuits with one or more nonlinear devices)

What analysis tools or methods can be used?

KCL ?

Nodal Analysis ?

KVL?

Mesh Analysis ?

~~Superposition?~~

Two-Port Subcircuits ?

~~Voltage Divider ?~~

~~Passing Current ?~~

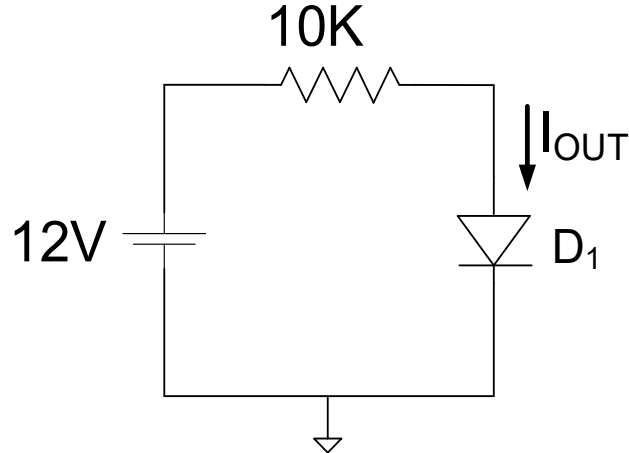
~~Current Divider?~~

~~Blocking Current ?~~

~~Thevenin and Norton Equivalent Circuits?~~

- How are piecewise models accommodated?
- Will address the issue of how to rigorously analyze nonlinear circuits with piecewise models later

Example: Determine I_{OUT} for the following circuit



$$I_{OUT} = I_S \left(e^{\frac{-I_{OUT} \cdot 10K}{V_t}} e^{\frac{12}{V_t}} - 1 \right)$$

- Results are accurate
- Analysis was tedious (and if slightly more complicated circuit even single implicit expression for output is often not attainable)
- Difficult to interpret results with implicit solution

Alternate Solution Strategy:

1. Assume PWL model with $V_D=0.6V$, $R_D=0$
2. Guess state of diode (ON)
3. Analyze circuit with model
4. Validate state of guess in step 2 (verify the “if” condition in model)

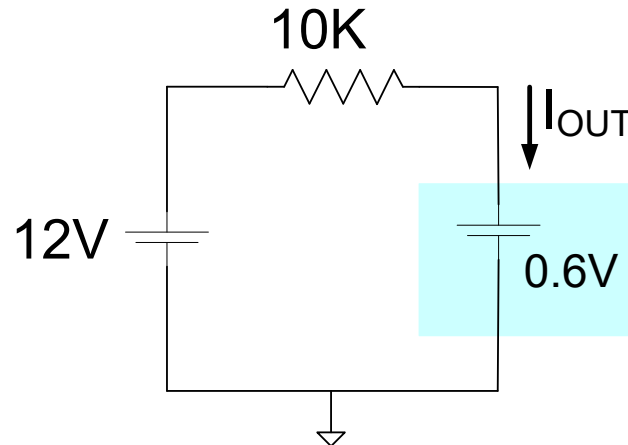
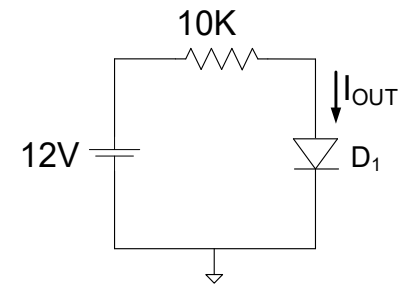
Select
Model

5. Assume PWL with $V_D=0.7V$
6. Guess state of diode (ON)
7. Analyze circuit with model
8. Validate state of guess in step 6 (verify the “if” condition in model)
9. Show difference between results using these two models is small
10. If difference is not small, must use a different model

Validate
Model

Alternate Solution:

1. Assume PWL model with $V_D=0.6V$, $R_D=0$, $I_S=10fA$
2. Guess state of diode (ON)



3. Analyze circuit with model

$$I_{OUT} = \frac{12V - 0.6V}{10K} = 1.14mA$$

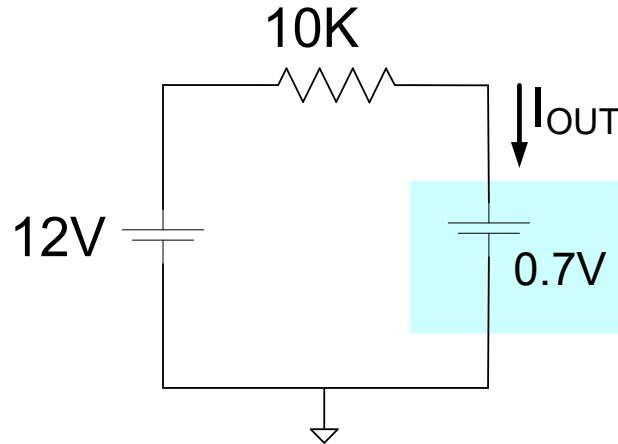
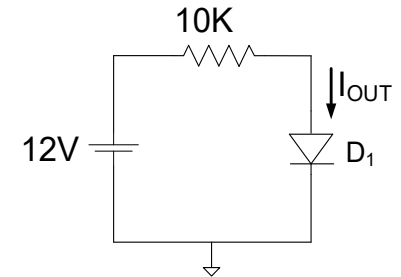
4. Validate state of guess in step 2

To validate state, must show $I_D > 0$

$$I_D = I_{OUT} = 1.14mA > 0$$

Alternate Solution:

5. Assume PWL model with $V_D=0.7V$, $R_D=0$, $I_S=10fA$
6. Guess state of diode (ON)



7. Analyze circuit with model

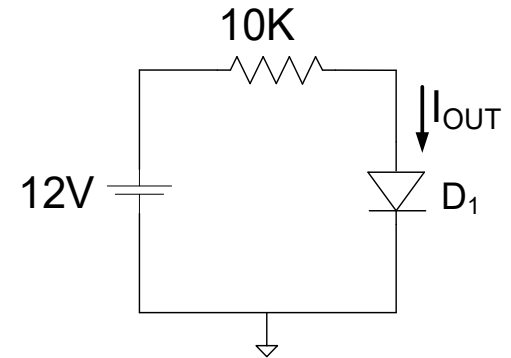
$$I_{OUT} = \frac{12V - 0.7V}{10K} = 1.13mA$$

8. Validate state of guess in step 6

To validate state, must show $I_D > 0$

$$I_D = I_{OUT} = 1.13mA > 0$$

Alternate Solution:



9. Show difference between results using these two models is small

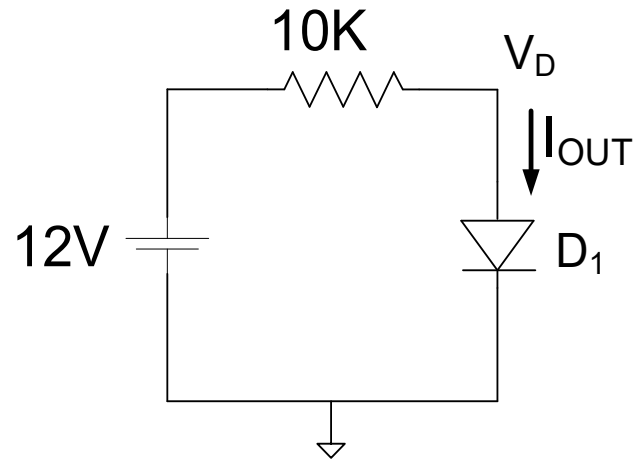
$I_{OUT} = 1.14\text{mA}$ and $I_{OUT} = 1.13\text{ mA}$ are close

Thus, can conclude

$$I_{OUT} \cong 1.14\text{mA}$$

Example:

Determine I_{OUT} for the following circuit



How do the two solutions compare?

With diode equation model for $I_S = 10\text{fA}$:

$$I_{OUT} = I_S \left(e^{\frac{-I_{OUT} \cdot 10K}{V_t}} e^{\frac{12}{V_t}} - 1 \right) \Rightarrow I_{OUT} = 1.134\text{mA}$$

With PWL model:

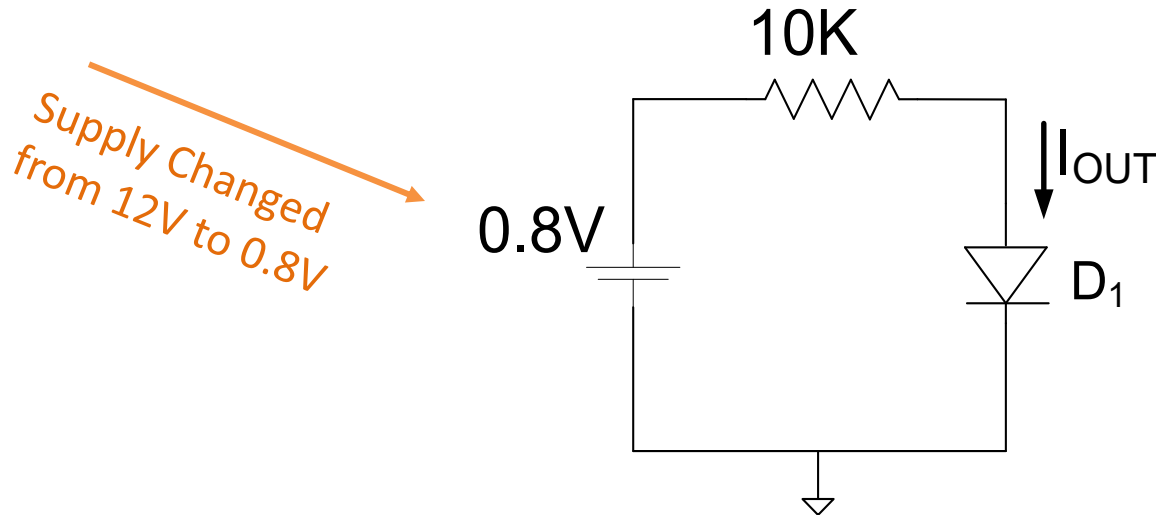
$$I_{OUT} \cong 1.14\text{mA}$$

What was the major reason the PWL model simplified the analysis?

Piecewise Linear Model

Example:

Determine I_{OUT} for the following circuit



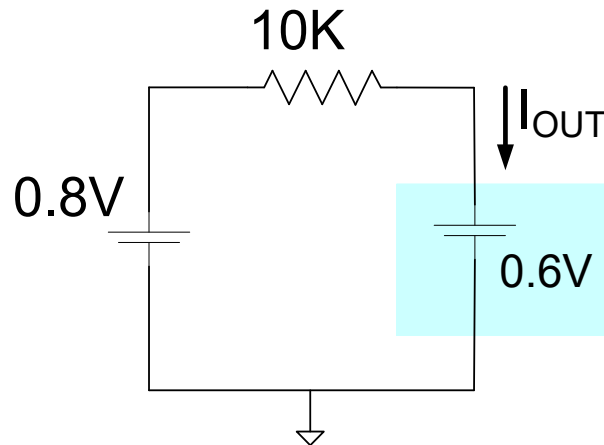
Solution:

Strategy:

1. Assume PWL model with $V_D=0.6V$, $R_D=0$
2. Guess state of diode (ON)
3. Analyze circuit with model
4. Validate state of guess in step 2
5. Assume PWL with $V_D=0.7V$
6. Guess state of diode (ON)
7. Analyze circuit with model
8. Validate state of guess in step 6
9. Show difference between results using these two models is small
10. If difference is not small, must use a different model

Solution:

1. Assume PWL model with $V_D=0.6V$, $R_D=0$
2. Guess state of diode (ON)



3. Analyze circuit with model

$$I_{OUT} = \frac{0.8 - 0.6V}{10K} = 20\mu A$$

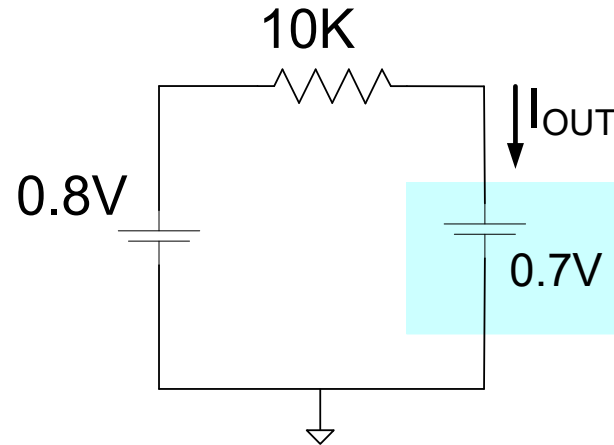
4. Validate state of guess in step 2

To validate state, must show $I_D > 0$

$$I_D = I_{OUT} = 20\mu A > 0$$

Solution:

5. Assume PWL model with $V_D=0.7V$, $R_D=0$
6. Guess state of diode (ON)



7. Analyze circuit with model

$$I_{OUT} = \frac{0.8V - 0.7V}{10K} = 10\mu A$$

8. Validate state of guess in step 6

To validate state, must show $I_D > 0$

$$I_D = I_{OUT} = 10\mu A > 0$$

Solution:

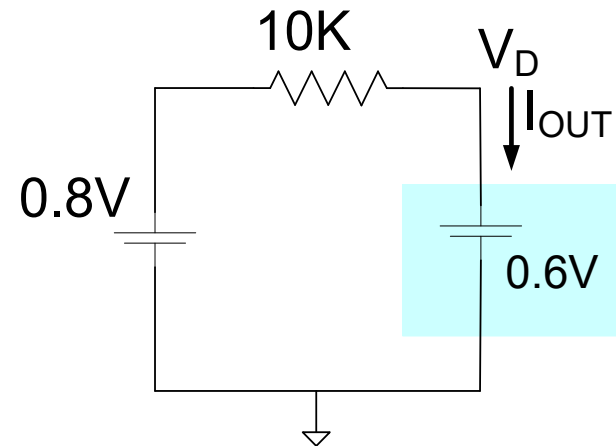
9. Show difference between results using these two models is small

$$I_{OUT}=10\mu A \text{ and } I_{OUT}=20\mu A \quad \text{are not close}$$

10. If difference is not small, must use a different model

Thus must use diode equation to model the device

$$\left. \begin{aligned} I_{OUT} &= \frac{0.8 - V_D}{10K} \\ I_{OUT} &= I_S e^{\frac{V_D}{V_t}} \end{aligned} \right\}$$



Solve simultaneously, assume $V_t=25\text{mV}$, $I_S=1\text{fA}$

Solving these two equations by iteration, obtain $V_D = 0.6148\text{V}$ and $I_{OUT} = 18.60\mu\text{A}$

Use of Piecewise Models for Nonlinear Devices when Analyzing Electronic Circuits

Process:

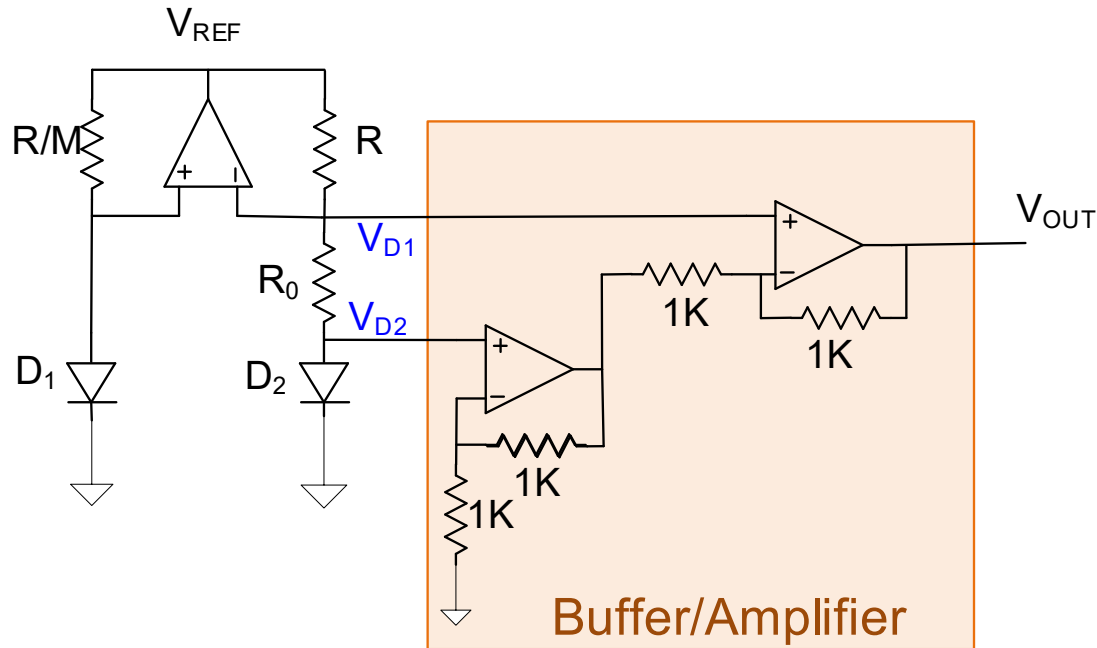
1. Guess state of the device
2. Analyze circuit
3. Verify State
4. Repeat steps 1 to 3 if verification fails
5. Verify model (if necessary)

Observations:

- Analysis generally simplified dramatically (particularly if piecewise model is linear)
- Approach applicable to wide variety of nonlinear devices
- Usually much faster than solving the nonlinear circuit directly
- Wrong guesses in the state of the device do not compromise solution (verification will fail)
- Helps to guess right the first time
- Detailed model is often not necessary with most nonlinear devices
- Particularly useful if piecewise model is PWL (but not necessary)
- Closed-form solutions (attainable with PWL models) give insight into performance of circuit
- For practical circuits, the simplified approach with piecewise models usually applies

Key Concept For Analyzing Circuits with Nonlinear Devices

A Diode Application



If buffer/amplifier added, serves as temperature sensor at V_{OUT}

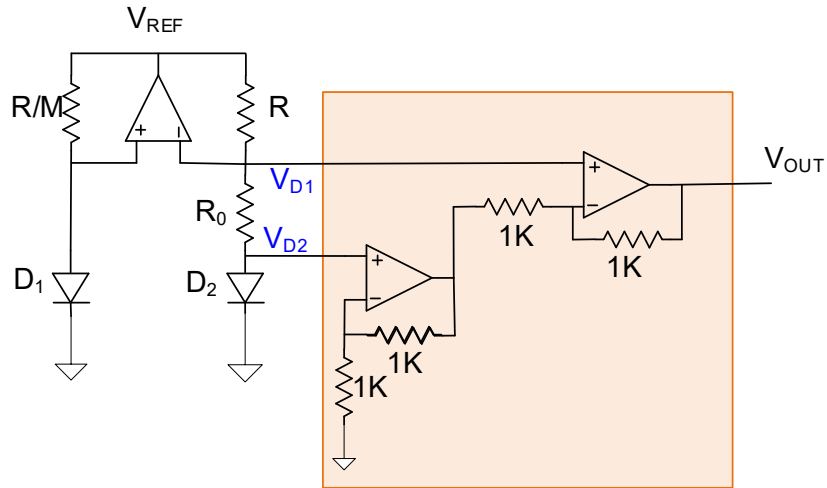
$$V_{OUT} = 2(V_{D1} - V_{D2})$$

May need compensation and startup circuits

For appropriate R_0 , serves as bandgap voltage reference at V_{REF} (buffer/amplifier excluded)

$$V_{REF} = V_{D1} + \frac{R}{R_0}(V_{D1} - V_{D2})$$

A Diode Application



$$V_{OUT} = 2(V_{D1} - V_{D2})$$

Analysis of temperature sensor (assume D_1 and D_2 matched)

$$\left. \begin{aligned} I_{D2}(T) &= \left(J_{SX} \left[T^m e^{\frac{-V_{G0}}{V_t}} \right] \right) A e^{\frac{V_{D2}}{V_t}} \\ I_{D1}(T) &= \left(J_{SX} \left[T^m e^{\frac{-V_{G0}}{V_t}} \right] \right) A e^{\frac{V_{D1}}{V_t}} \\ I_{D1}(T) &= M I_{D2}(T) \end{aligned} \right\}$$

$$V_t = \frac{k}{q} T$$



$$\left(J_{SX} \left[T^m e^{\frac{-V_{G0}}{V_t}} \right] \right) A e^{\frac{V_{D1}}{V_t}} = M \left(J_{SX} \left[T^m e^{\frac{-V_{G0}}{V_t}} \right] \right) A e^{\frac{V_{D2}}{V_t}}$$

Cancelling terms and taking ln we obtain

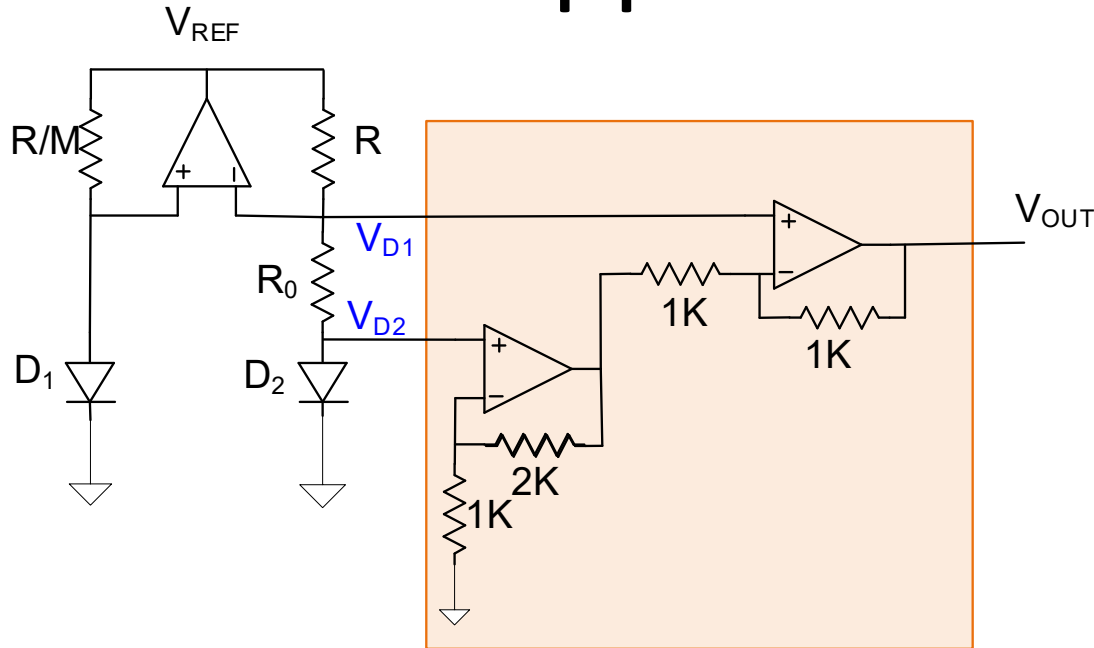
$$V_{D1} - V_{D2} = V_t \ln M$$

Thus

$$V_{OUT} = 2(V_{D1} - V_{D2}) = 2 \ln M \cdot \frac{k}{q} T$$

$$T = V_{OUT} \frac{q}{2k \ln M}$$

A Diode Application



May need compensation and startup circuits

If buffer/amplifier added, serves as temperature sensor at V_{OUT}

$$V_{OUT} = 2(V_{D1} - V_{D2}) \quad \Rightarrow \quad T = V_{OUT} \frac{q}{2k \ln M}$$

For appropriate R_0 , serves as bandgap voltage reference

$$V_{REF} = V_{D1} + \frac{R}{R_0}(V_{D1} - V_{D2}) \quad \Rightarrow \quad ?$$

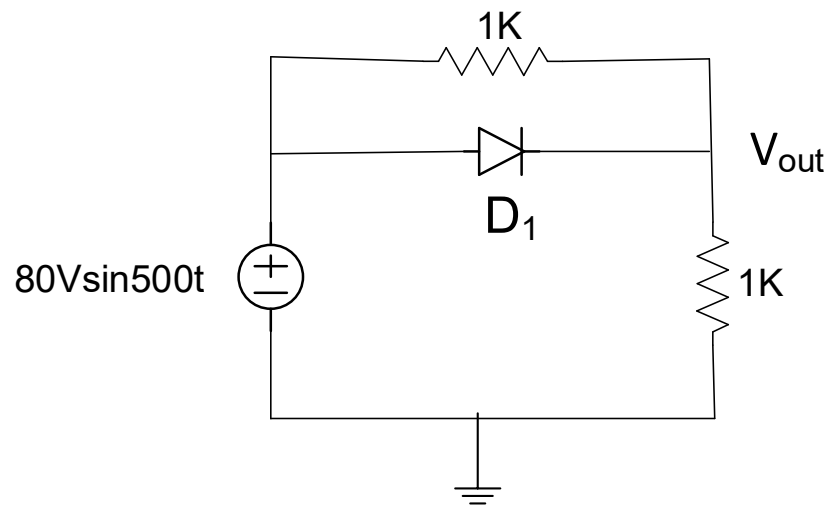
Analysis of V_{REF} to show output is nearly independent of T and V_{DD} is more tedious

Use of Piecewise Models for Nonlinear Devices when Analyzing Electronic Circuits

Process:

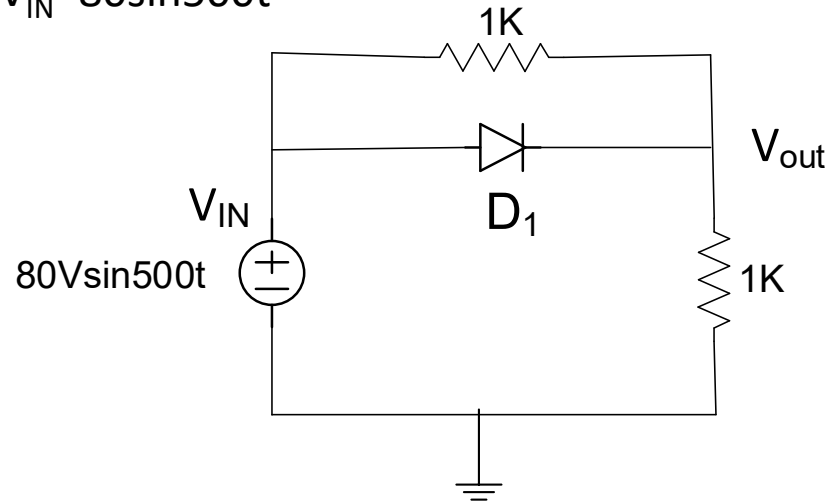
1. Guess state of the device
2. Analyze circuit
3. Verify State
4. Repeat steps 1 to 3 if verification fails
5. Verify model (if necessary)

What about nonlinear circuits (using piecewise models) with time-varying inputs?

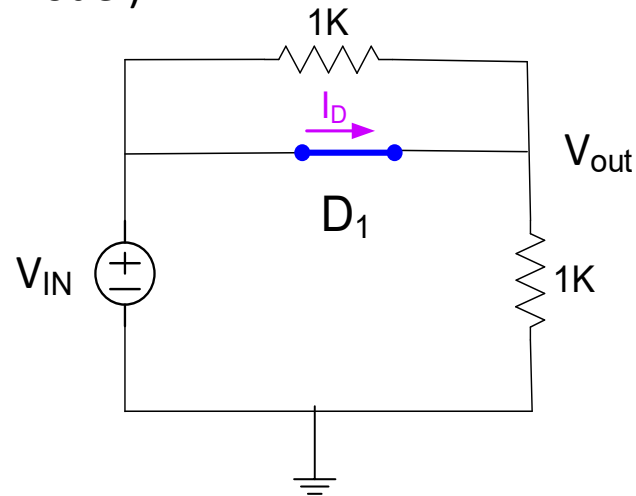


Same process except state verification (step 3) may include a range where solution is valid

Example: Determine V_{OUT} for $V_{IN}=80\sin 500t$



Guess D_1 ON (will use ideal diode model)

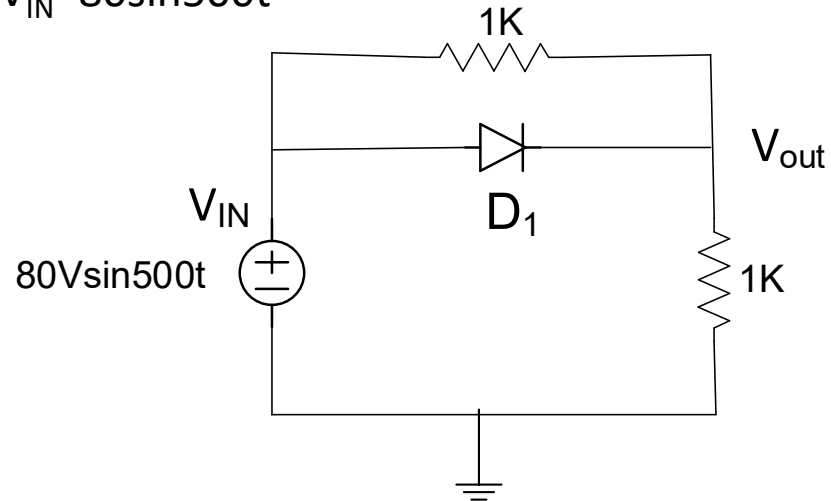


$$V_{OUT}=V_{IN}=80\sin(500t)$$

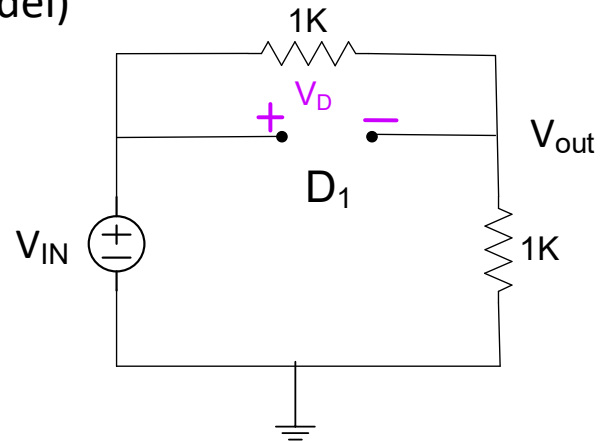
Valid for $I_D > 0$
$$I_D = \frac{V_{IN}}{1K}$$

Thus valid for $V_{IN} > 0$

Example: Determine V_{OUT} for $V_{IN}=80\sin 500t$



Guess D_1 OFF (will use ideal diode model)

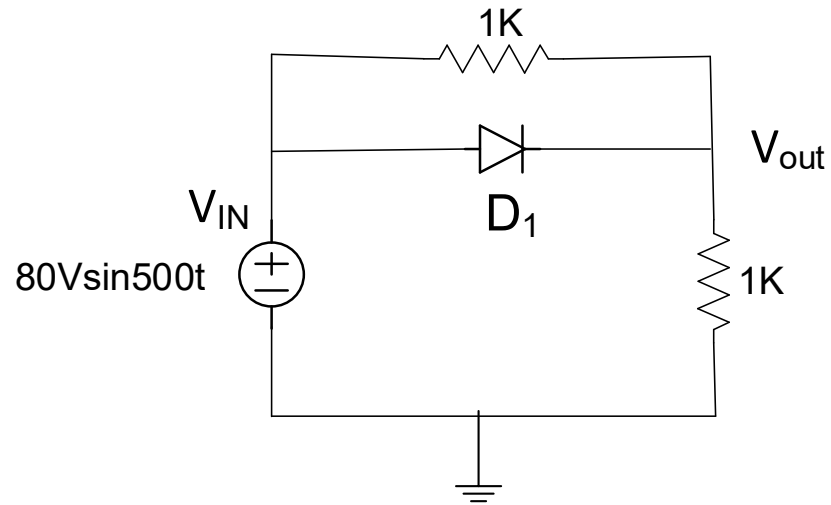


$$V_{OUT}=V_{IN}/2=40\sin(500t)$$

Valid for $V_D < 0$ $V_D = \frac{V_{IN}}{2}$

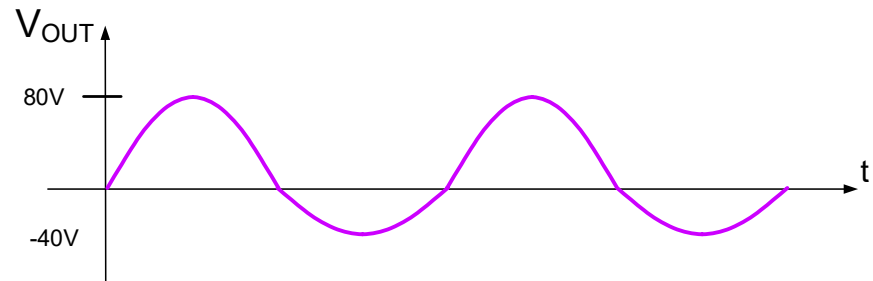
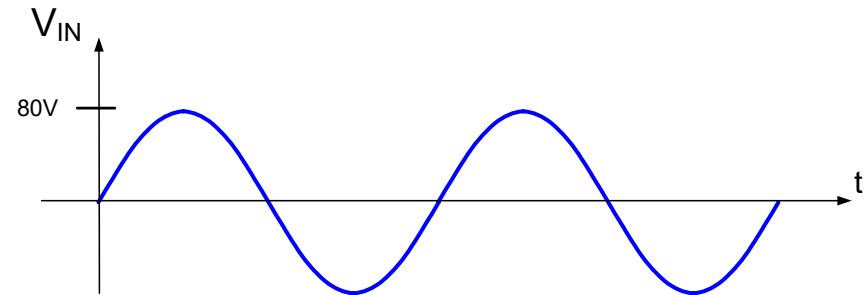
Thus valid for $V_{IN} < 0$

Example: Determine V_{OUT} for $V_{IN}=80\sin 500t$



Thus overall solution

$$V_{OUT} = \begin{cases} 80 \sin 500t & \text{for } V_{IN} > 0 \\ 40 \sin 500t & \text{for } V_{IN} < 0 \end{cases}$$

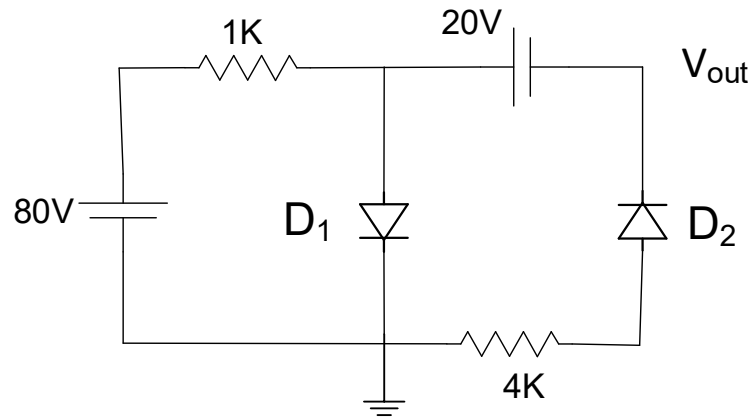


Use of Piecewise Models for Nonlinear Devices when Analyzing Electronic Circuits

Process:

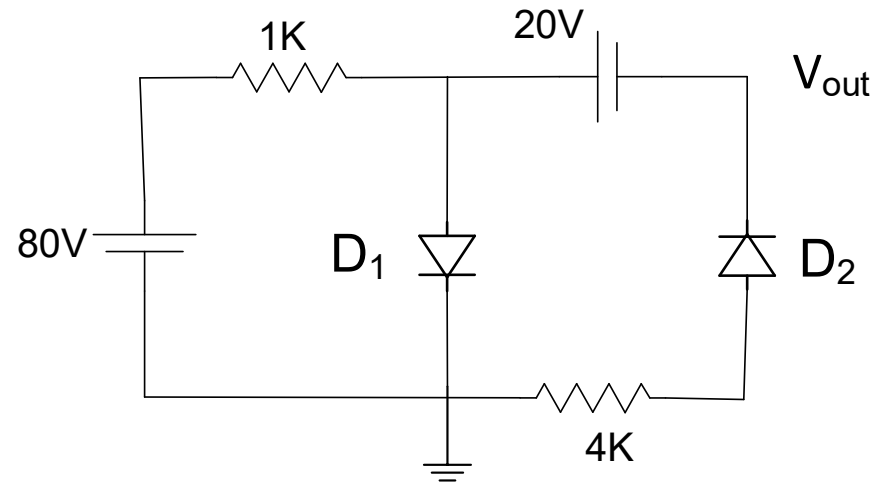
1. Guess state of the device
2. Analyze circuit
3. Verify State
4. Repeat steps 1 to 3 if verification fails
5. Verify model (if necessary)

What about circuits (using piecewise models) with multiple nonlinear devices?

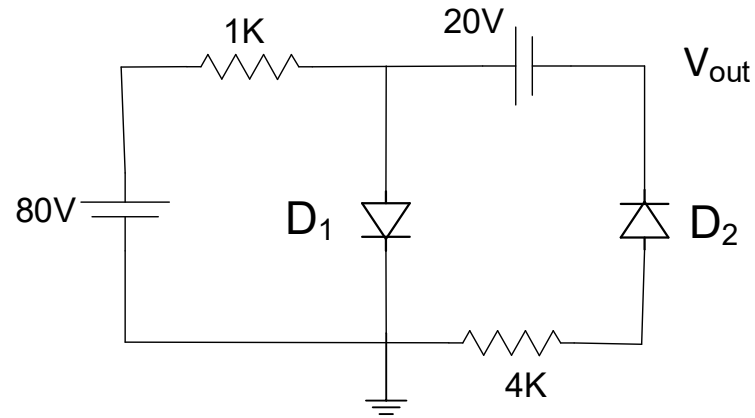


Guess state for each device (multiple combinations possible)

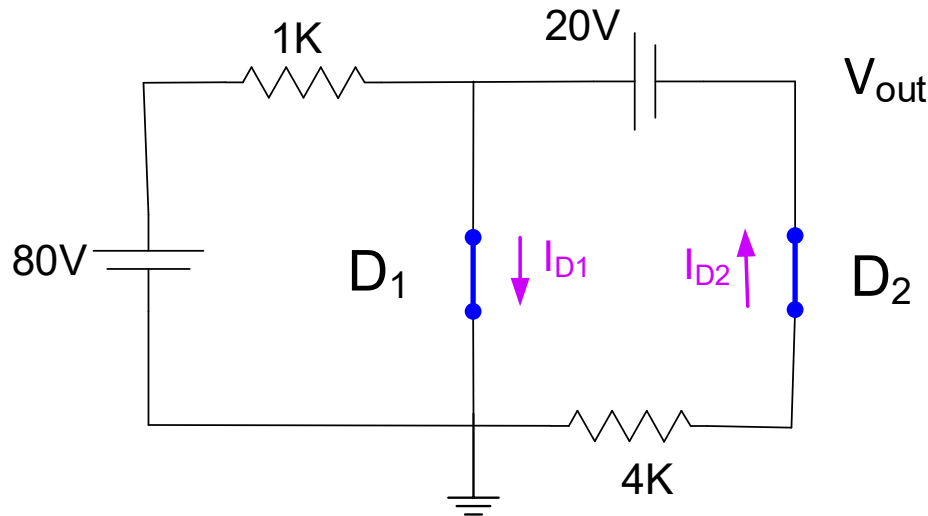
Example: Obtain V_{OUT}



Example: Obtain V_{OUT}



Guess D_1 and D_2 on



$$V_{OUT} = -20V$$

Valid for $I_{D2} > 0$ and $I_{D1} > 0$

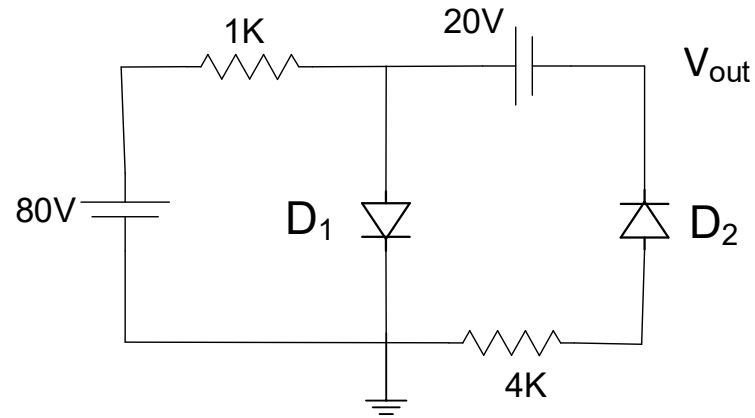
$$I_{D2} = \frac{20V}{4K} = 5mA > 0 \quad I_{D1} = \frac{80V}{1K} + I_{D2} = 85mA > 0$$

Validates

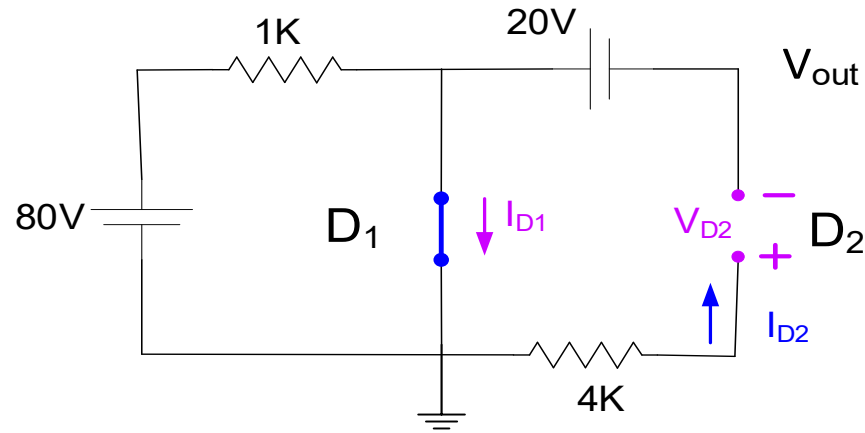
Validates

Since validates, solution is valid

Example: Obtain V_{OUT}



If we had guessed wrong
Guess D_1 ON and D_2 OFF



$$V_{OUT} = -20V$$

Valid for $I_{D1} > 0$ and $V_{D2} < 0$

$$I_{D1} = \frac{80V}{1K} = 80mA > 0$$

$$V_{D2} = +20$$

Validates

Fails
Validation

Since fails to validate, solution is not valid so guess is wrong !

Use of Piecewise Models for Nonlinear Devices when Analyzing Electronic Circuits

Single Nonlinear Device

Process:

1. Guess state of the device
2. Analyze circuit
3. Verify State
4. Repeat steps 1 to 3 if verification fails
5. Verify model (if necessary)

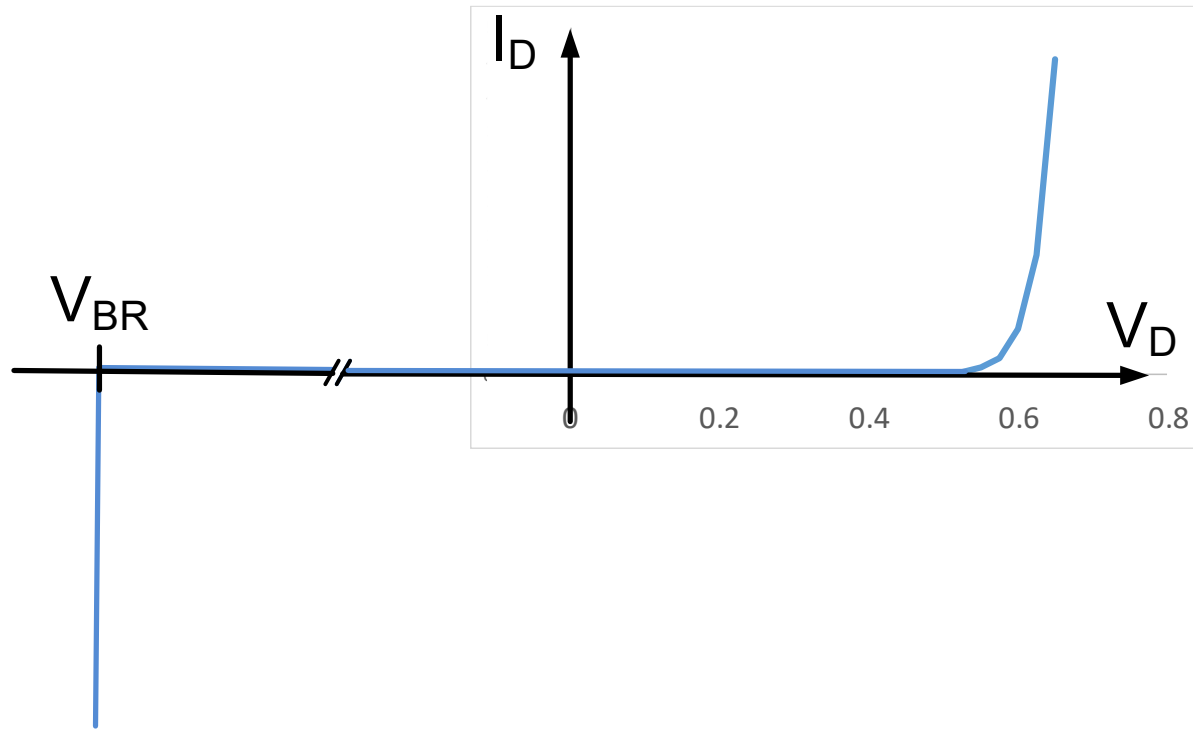
Multiple Nonlinear Devices

Process:

1. Guess state of each device (may be multiple combinations)
2. Analyze circuit
3. Verify State
4. Repeat steps 1 to 3 if verification fails
5. Verify models (if necessary)

Analytical solutions of circuits with multiple nonlinear devices are often impossible to obtain if detailed non-piecewise nonlinear models are used

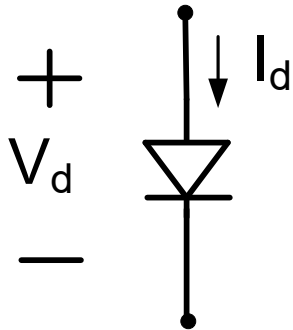
Diode Breakdown



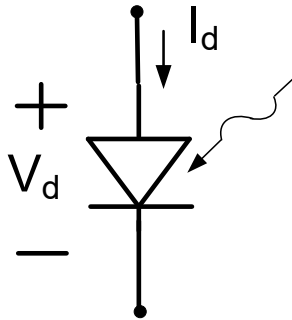
- Diodes will “break down” if a large reverse bias is applied
- Unless current is limited, reverse breakdown is destructive
- Breakdown is very sharp
- For many signal diodes, V_{BR} is in the -100V to -1000V range
- Relatively easy to design circuits so that with correct diodes, breakdown will not occur
- Zener diodes have a relatively small breakdown and current is intentionally limited to use this breakdown to build voltage references

Types of Diodes

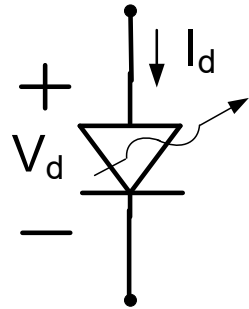
pn junction diodes



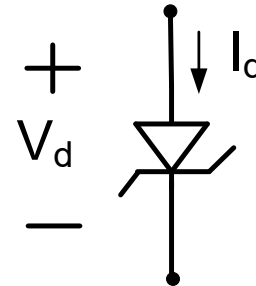
Signal or Rectifier



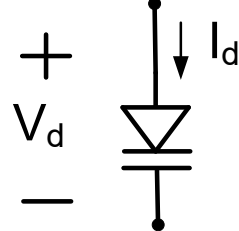
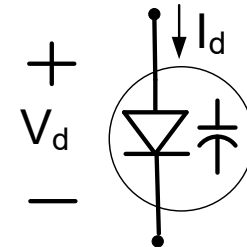
Pin or Photo



Light Emitting LED
Laser Diode

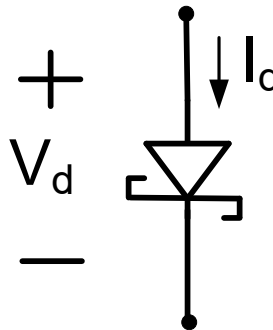
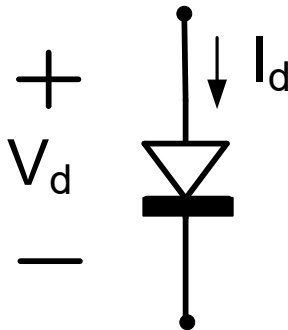


Zener

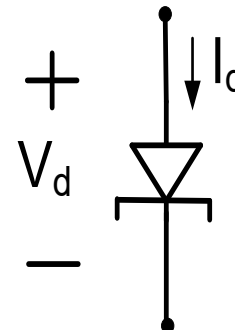


Varactor or Varicap

Metal-semiconductor junction diodes



Schottky Barrier



Basic Devices and Device Models

- Resistor
- Diode

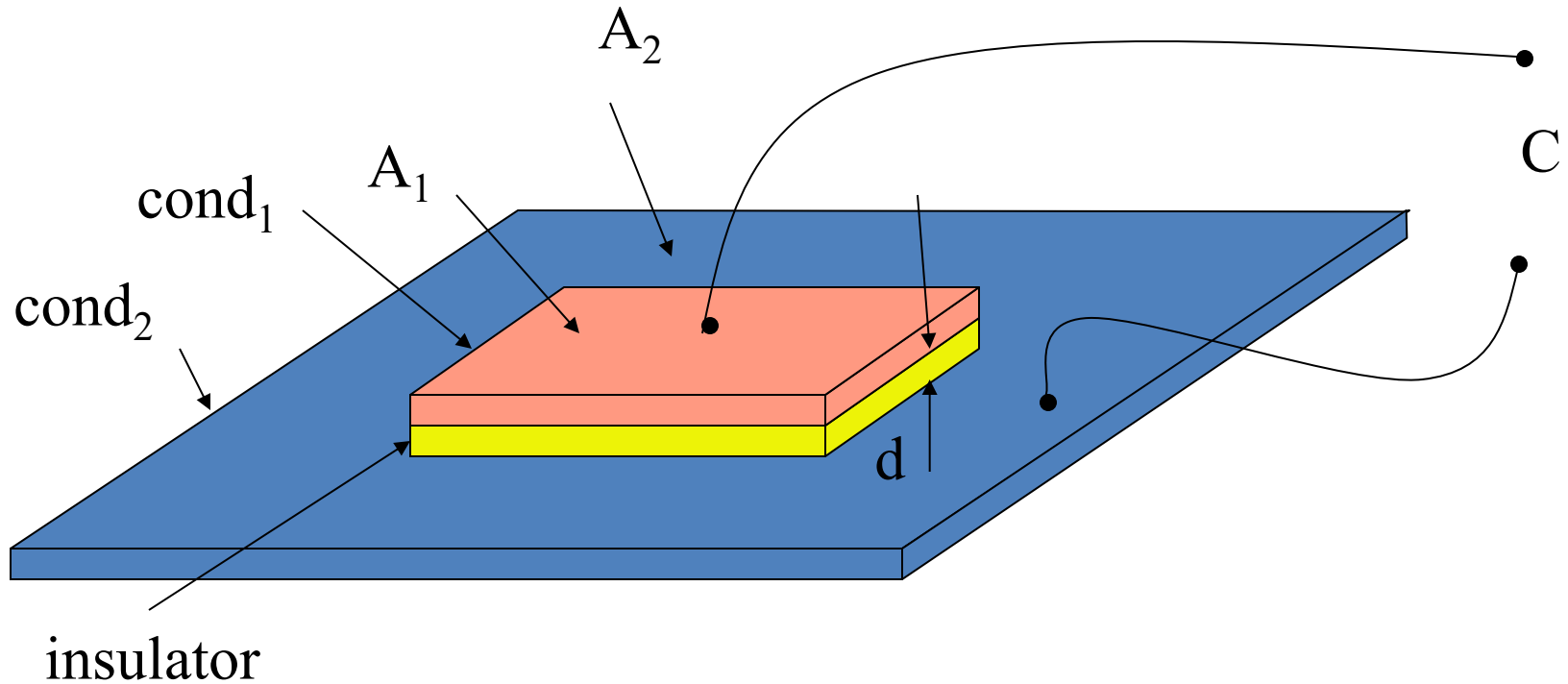
 Capacitor

- MOSFET
- BJT

Capacitors

- Types
 - Parallel Plate
 - Fringe
 - Junction

Parallel Plate Capacitors



A = area of intersection of A_1 & A_2

One (top) plate **intentionally** sized smaller to determine C

$$C = \frac{\epsilon A}{d}$$

Parallel Plate Capacitors

$$\text{If } C_d = \frac{\text{Cap}}{\text{unit area}}$$

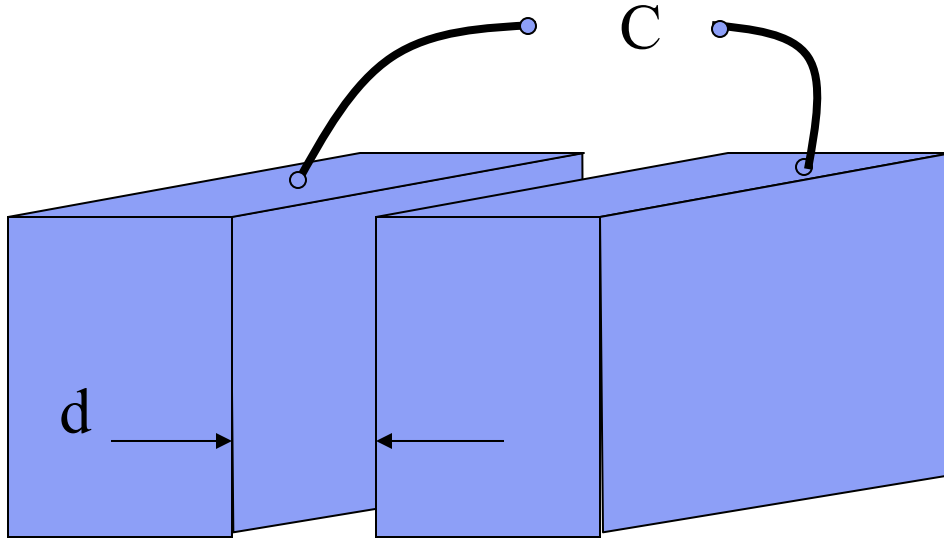
$$C = \frac{\epsilon A}{d}$$

$$C = C_d A$$

where

$$C_d = \frac{\epsilon}{d}$$

Fringe Capacitors

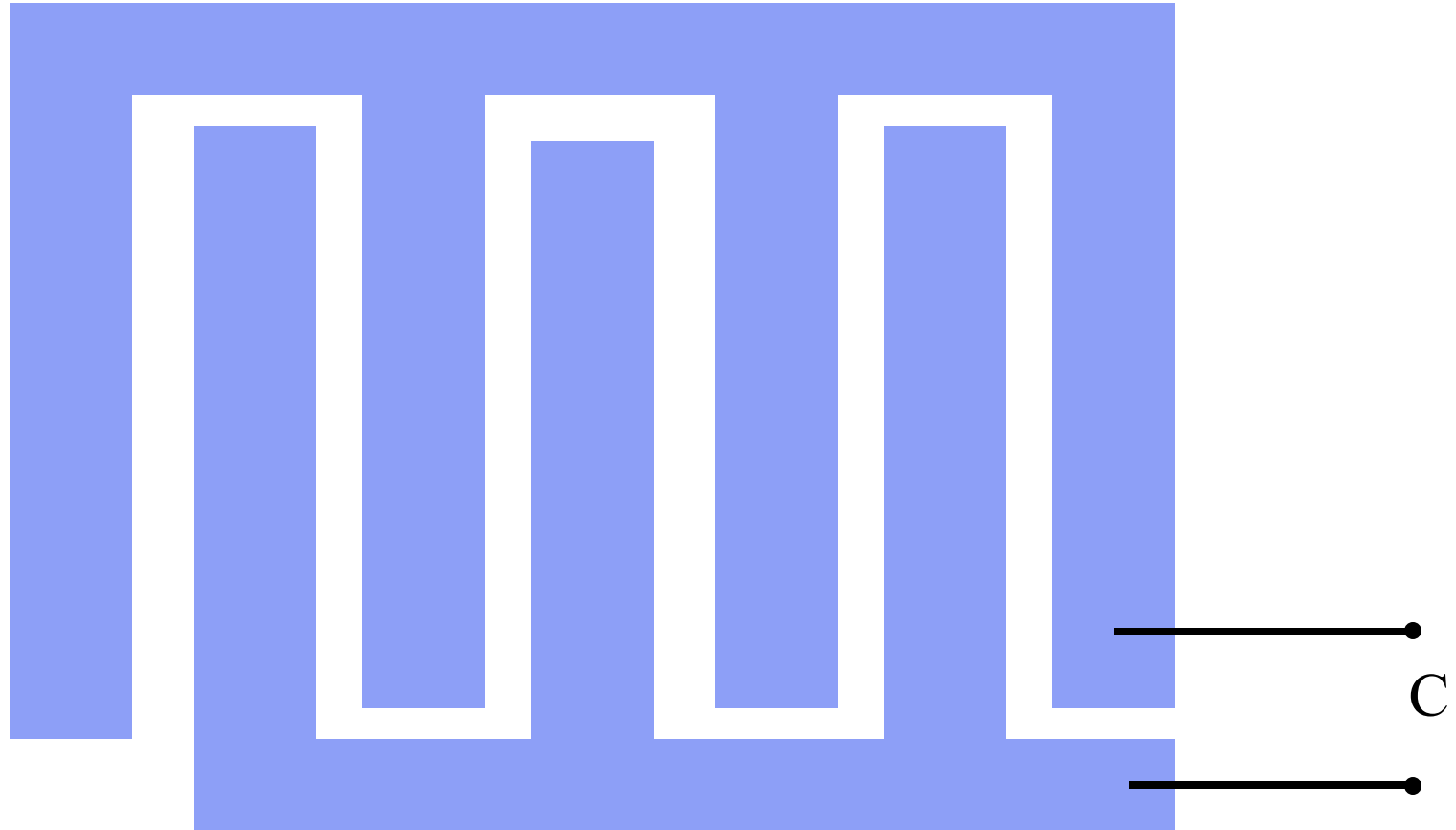


$$C = \frac{\epsilon A}{d}$$

A is the area where the two plates are parallel

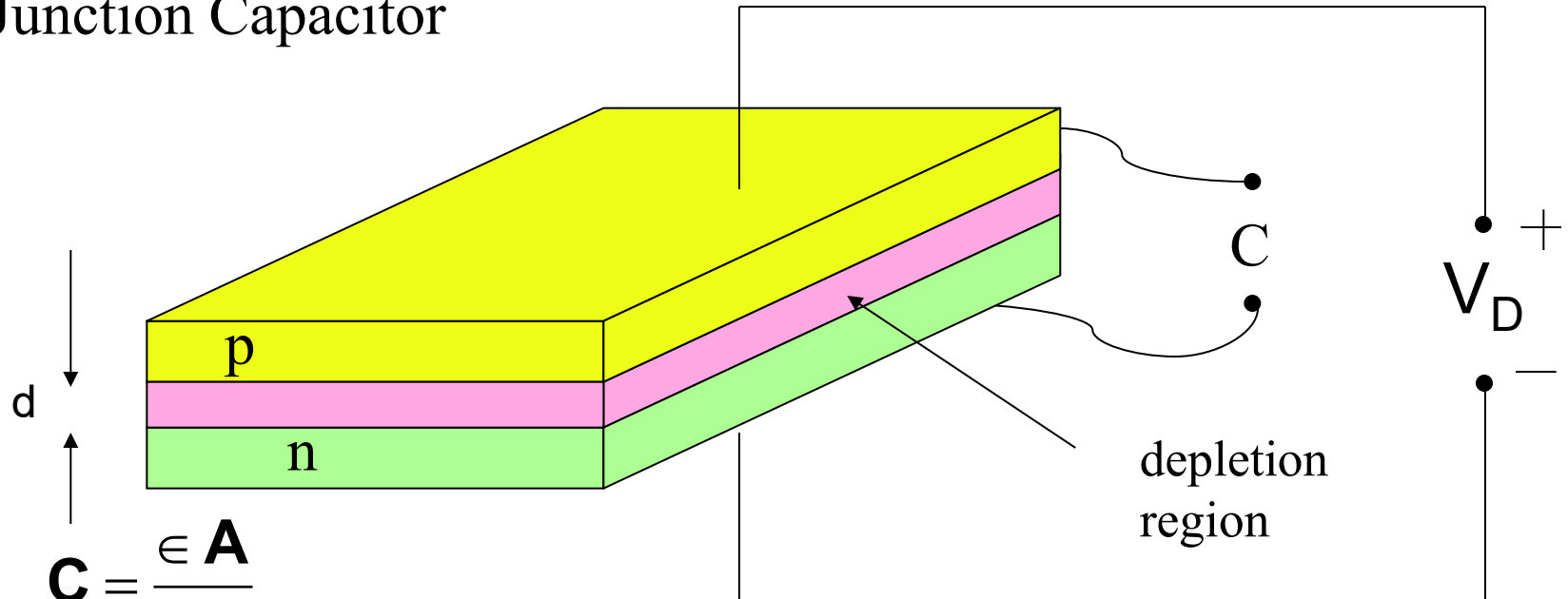
Only a single layer is needed to make fringe capacitors

Fringe Capacitors



Capacitance

Junction Capacitor



$$C = \frac{\epsilon A}{d}$$

ϵ is dielectric constant

$$C = \frac{C_{j0} A}{\left(1 - \frac{V_D}{\phi_B}\right)^n} \quad \text{for } V_{FB} < \frac{\phi_B}{2}$$

Note: d is voltage dependent

-capacitance is voltage dependent

-usually parasitic caps

-varicaps or varactor diodes exploit voltage dep. of C

C_{j0} is the zero—bias junction capacitance density

Model parameters $\{C_{j0}, n, \phi_B\}$ Design parameters $\{A\}$

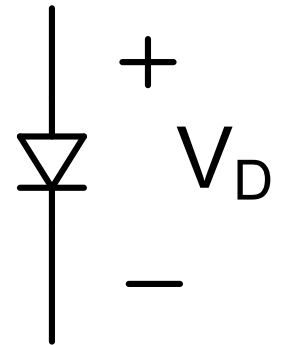
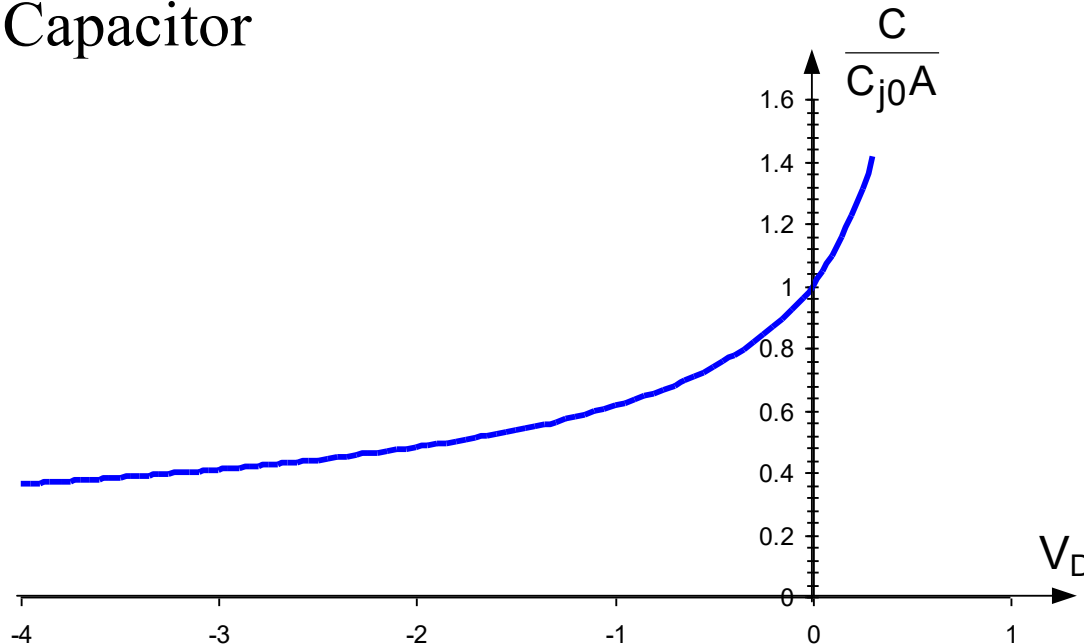
$$\phi_B \cong 0.6V$$

$$n \simeq 0.5$$

$$C_{j0} \text{ highly process dependent around } 500\text{aF}/\mu\text{m}^2$$

Capacitance

Junction Capacitor



$$C = \frac{C_{j0}A}{\left(1 - \frac{V_D}{\phi_B}\right)^n} \quad \text{for } V_{FB} < \frac{\phi_B}{2}$$

Voltage dependence is substantial

$$\phi_B \cong 0.6V \quad n \cong 0.5$$



Stay Safe and Stay Healthy !

End of Lecture 15